IN 221 (AUG) 3:0
Sensors and Transducers
Electromagnetic Sensors and Transducers
Lecture 2

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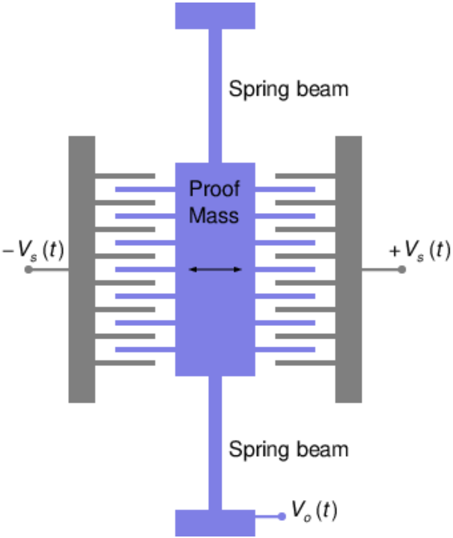
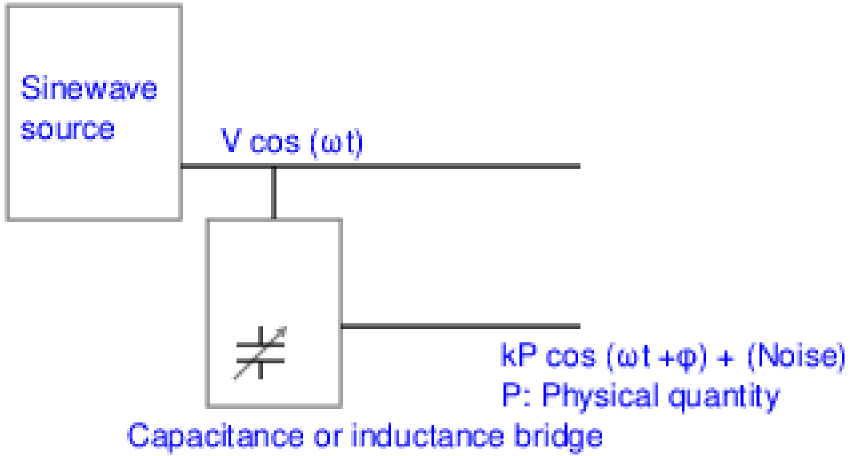
Accelerometer: Interdigitated Structure



Accelerometer or Pressure sensor

- A flexible electrode moves a small distance that is proportional to the acceleration or the pressure difference to be measured.
- This small movement changes a capacitance or an inductance.
- With a bridge arrangement, this sensor can be ...
  - ... sensitive,
  - ... reliable,
  - ... and quite linear.

How do we measure *P*?



How do we measure P?

Physical quantity to be measured: P
Output: (Signal) + (Noise)
Signal: $kP\cos(\omega t + \phi)$

$$v_o = kP\cos(\omega t + \phi) + (\text{Noise})$$

- The signal output may be quite small.
- It is still proportional to P.
- Here, ϕ is a constant phase shift that is specific to the bridge instrumentation.

Trouble with simple amplification

- In industrial settings, the noise strength may be comparable to the signal. It may sometimes be greater.
- If we simply amplify the output, the noise will also be amplified.
- Filtering will decrease the noise, but that may not be enough.
- Cases like this are very common.

Lock-in Amplifier

- Solution: Use a *lock-in amplifier*.
- Other names:
  - Synchronous detection
  - Phase-sensitive detection
- Implementation: Multiply the bridge output v_o with the reference sinewave and average (low-pass filter) it.
- What is the resulting output?

Lock-in Amplifier Mathematics

The output of the multiplier is

$$V\cos(\omega t) \times kP\cos(\omega t + \phi) = kVP[\cos(\phi)\cos^2(\omega t) - \sin(\phi)\sin(\omega t)\cos(\omega t)]. \qquad (1)$$

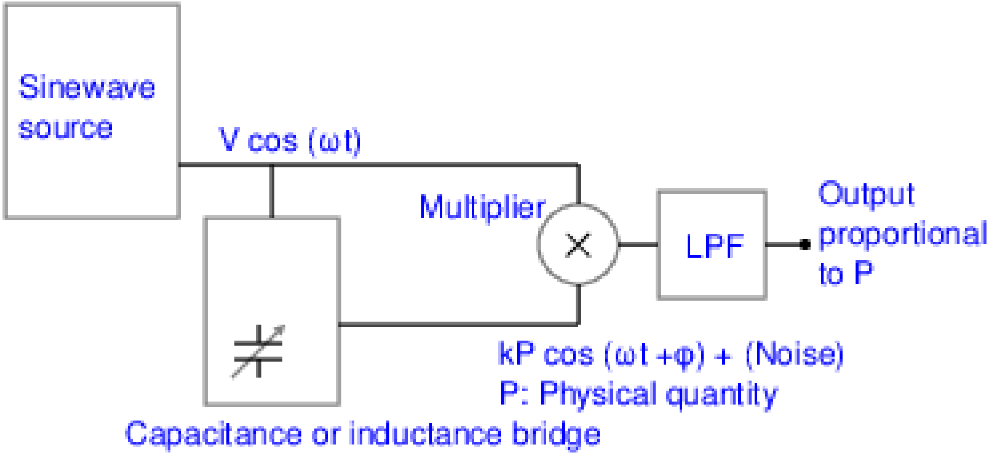
$$\cos^2(\omega t) = \frac{1}{2} + \frac{\cos(2\omega t)}{2}, \qquad (2)$$

and

$$\sin(\omega t)\cos(\omega t) = \frac{\sin(2\omega t)}{2}. \qquad (3)$$

Both $\cos(2\omega t)$ and $\sin(2\omega t)$ have average value of 0.
So the average value of $\cos^2(\omega t)$ is $\frac{1}{2}$, and that of $\sin(\omega t)\cos(\omega t)$ is 0.
The average value of the product is $\frac{1}{2}kVP\cos(\phi)$. $\Rightarrow$ (LPF output) $\propto P$.
(If we are unlucky to have $\cos\phi = 0$, we can use $V\sin(\omega t)$ instead of $V\cos(\omega t)$ at one input of the multiplier.)

Lock-in Amplifier: Block diagram

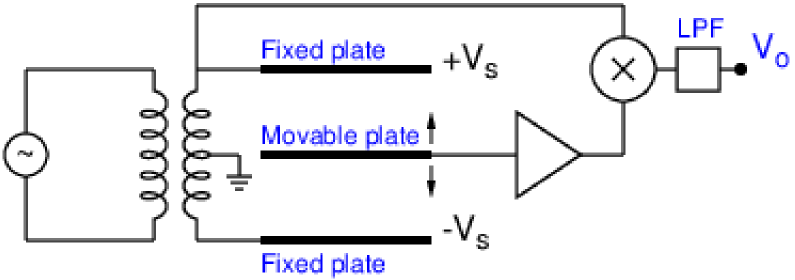


Demonstration of a Lock-in Amplifier

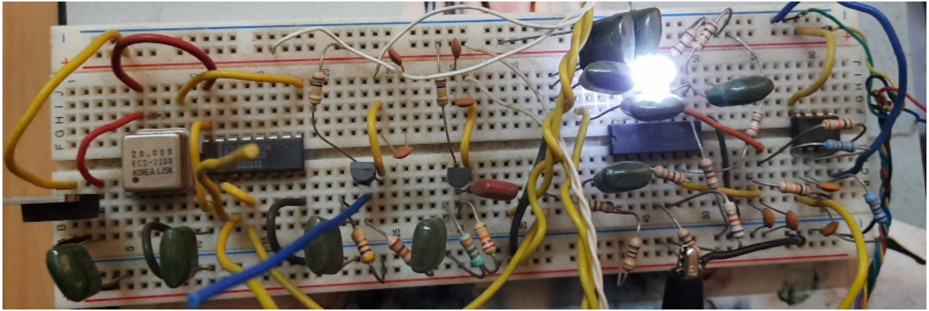
The following circuit shows how we may measure the difference between two small capacitances using a lock-in amplifier.

- Constructed on bread board.
- $\Delta C < 1\,\mathrm{pF}$
- Layout not neat at all ...
- ... yet the output signal is very clean.

Capacitive Sensor: Block Diagram



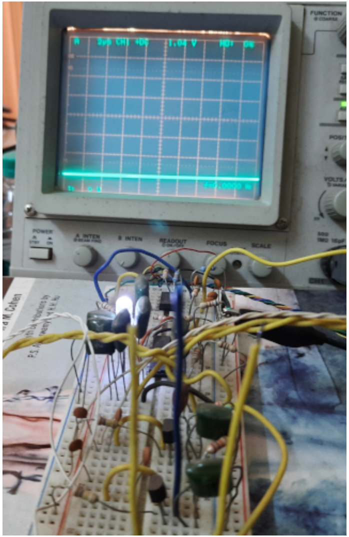
Example Differential Capacitive Sensor



Capacitive Sensor: Circuit Details

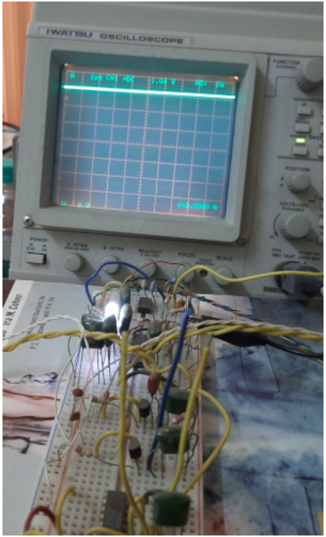
- 20 MHz crystal oscillator output given to a divide by 2 counter.
- 1/4 of 74LS175 D flip-flop used to implement the divide by 2 counter.
- Q_0 output is $+V_s(t)$, $\overline{Q}_0$ output is $-V_s(t)$
- The two yellow wires sticking out are the fixed plates.
- The blue wire sticking out is the movable plate.
- The amplifier is a two-transistor tuned amplifier.
- MC1496 is used as the multiplier.
- Uses RC LPF.
- NE5532 is the final amplifier after the LPF.

Blue Wire Closer to the Left Yellow Wire



Blue Wire Equidistant from the Yellow Wires

Blue Wire Closer to the Right Yellow Wire



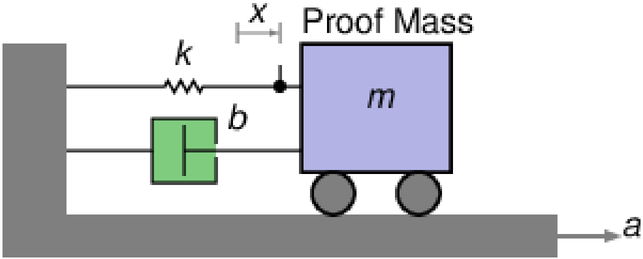
MEMS Accelerometer

What is an accelerometer?

- A sensor whose output is proportional to its acceleration, at least when the acceleration is changing slowly
- If the acceleration is changing too fast, then the output may not be able to keep up with the input.
- Accelerometers usually have three output channels for the three components of acceleration.
- Some accelerometers provide analogue voltage outputs, while others internally convert their outputs to a digital form.
- Accelerometers made using MEMS technology are very successful.

Accelerometer: Applications

- Sensing the acceleration of aircraft and vehicles
- Sensing the orientation of hand-held devices using the acceleration due to gravity
- Sensing vibration in industrial applications



Simplified Mass-Spring-Dashpot model
x: Displacement of the mass from its equilibrium position in the frame of the accelerometer

$$m\ddot{x} = -kx - b\dot{x} - ma$$

$$m\ddot{x} + b\dot{x} + kx = -ma \tag{4}$$

Accelerometer: Transfer Function

$$T(s) = \frac{X(s)}{A(s)} = \frac{-1}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

Note: The negative sign is due to my sign convention.
Many books will indicate x in the reverse direction to not have this negative sign.

$$T(s) = \frac{-1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}, \tag{5}$$

where,

$$\omega_0 = \sqrt{\frac{k}{m}}, \tag{6}$$

and

$$Q = \frac{\sqrt{km}}{b}. \tag{7}$$

Accelerometer: Low Frequency Response

For $\omega \ll \omega_0$,

$$T(j\omega) \approx \frac{-1}{\omega_0^2} = -\frac{m}{k}. \tag{8}$$

Typical value: $f_0 = \dfrac{\omega_0}{2\pi} = 50\,\text{kHz}$

Conclusions

- For frequencies which are small compared to the resonant frequency, the displacement is proportional to the acceleration.
- There is a need to make the electrical output proportional to x.

Accelerometer: Sensing the Displacement

Differential Capacitance Arrangement

- The proof mass acts as an electrode that moves between two other electrodes.
- It forms two capacitances that are equal when there is no displacement.
- The difference between the capacitances is proportional to the displacement, provided the displacement is small.

Challenge

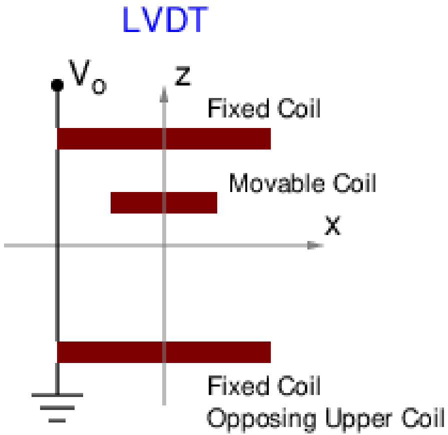
- The devices are very small.
- The change in capacitance is very small.
- How do we reliably detect this small change?
- Answer: Use lock-in amplifier.

LVDT

Now we discuss the LVDT, a displacement sensor.

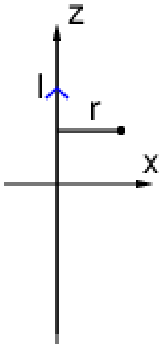
- Linear Variable Differential Transformer
- Two fixed coils in anti-series (in opposition)
- One movable coil
- Usually the movable coil is excited with sinewave AC.
- Fixed coil output is given to the phase sensitive detector.
- Quite robust and accurate

LVDT Coils



LVDT
V_o
z
Fixed Coil
Movable Coil
x
Fixed Coil
Opposing Upper Coil

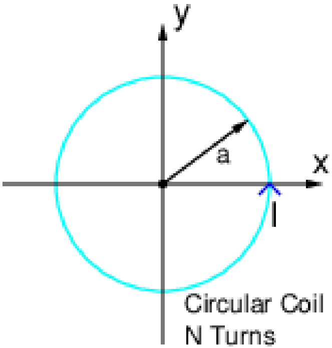
Magnetic Field due to Infinite Wire



$$B_\phi = \frac{\mu_0 I}{2\pi r} \tag{9}$$

Magnetic constant: $\mu_0 = 4\pi \times 10^{-7}$ T m / A
Example computation: If $I = 10\,\text{A}$, and $r = 1\,\text{cm}$, then $B_\phi = 0.2\,\text{mT}$.

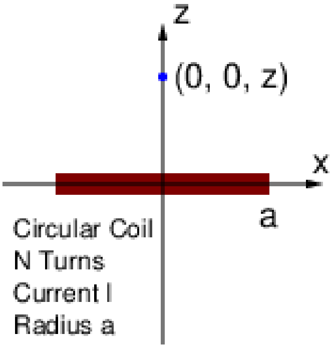
Circular Coil: B at Centre



$$B_z(0,0,0) = N\frac{\mu_0 I}{2a} \tag{10}$$

Example computation: If $N = 100$, $I = 1$ A, and $a = 5$ cm, then $B_z(0,0,0) = 1.2566$ mT.

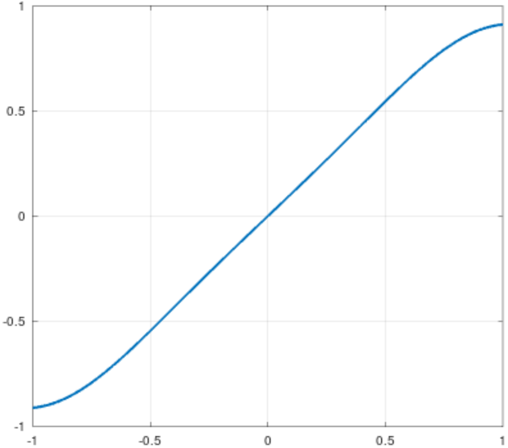
Circular Coil: B on the Axis



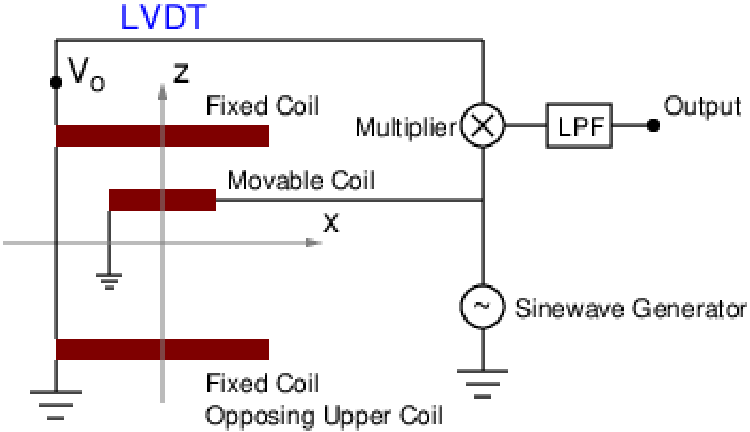
$$B_z(0,0,z) = N\frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}} \tag{11}$$

Example computation: If $N = 100$, $I = 1$ A, $a = 5$ cm, and $z = 5$ cm, then $B_z(0,0,z) = 0.444\,29$ mT.

LVDT Output Linearity



LVDT Block Diagram



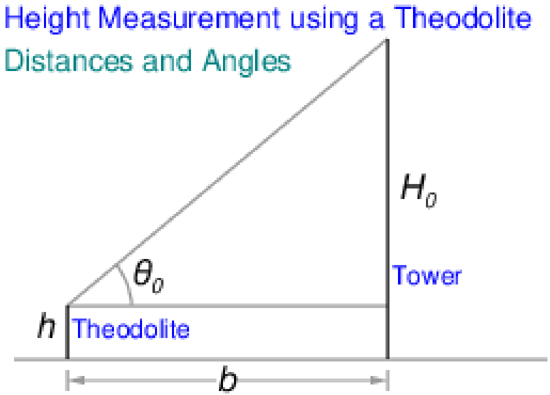
LVDT Analysis

Detailed analysis of the LVDT requires the study of the following concepts.

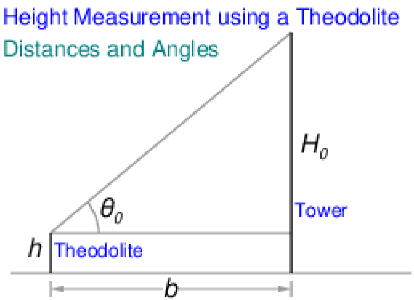
- Magnetic field of a coil
- Mutual inductance
- Lock-in amplifier

Example of Error Analysis

We wish to measure the height of a tower using a theodolite.



Another Example of Error Analysis

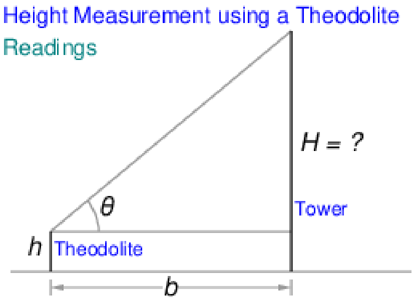


$$H_0 = h + b \tan \theta_0. \quad (12)$$

Note that

$$b = \frac{H_0 - h}{\tan \theta_0}. \quad (13)$$

Measurements



In actual measurement, the reading of the theodolite is θ, which is not the same as θ_0 due to instrument errors. Also, h and b can be measured very accurately. So we consider the angle (θ) measurement as the only source of error.

Estimated height is

$$H = h + b \tan \theta. \quad (14)$$

Error Analysis

Error in height is

$$H_0 - H = b(\tan\theta_0 - \tan\theta) \approx b\sec^2\theta_0 \times (\theta_0 - \theta).$$

But

$$b = \frac{H_0 - h}{\tan\theta_0}.$$

So

$$H_0 - H \approx b\sec^2\theta_0 \times (\theta_0 - \theta) = \frac{H_0 - h}{\tan\theta_0}\sec^2\theta_0 \times (\theta_0 - \theta).$$

Or,

$$H_0 - H \approx \frac{H_0 - h}{\sin\theta_0\cos\theta_0} \times (\theta_0 - \theta).$$

The error in angle measurement is magnified by a factor

$$K = \frac{H_0 - h}{\sin\theta_0\cos\theta_0} = 2\frac{H_0 - h}{\sin(2\theta_0)}. \tag{15}$$

What value of θ_0 minimizes K?

Answer

$\sin(2\theta_0)$ can be a maximum of 1, when $\theta_0 = \pi/4$.
So the answer is 45 degrees.
In other words, b should be as close to $H_0 - h$ as possible.