IN 221 (AUG) 3:0
Sensors and Transducers
Electromagnetic Sensors and Transducers
Lecture 3

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Temperature Sensors

- Thermistor
- Thermocouple
- Resistance Thermometer
- Silicon Bandgap Temperature Sensor
- Infrared Temperature Sensor

Thermistor

- Resistor whose resistance changes with temperature.
- Usually made of semiconductors.
- Change of resistance is more than that in metal resistors.
- Usually nonlinear.
- Limited range of use: -90 ℃ to 130 ℃
- Two types: NTC and PTC
- NTC: Negative Temperature Coefficient, resistance decreases with increase in temperature
- PTC: Positive Temperature Coefficient, resistance increases with increase in temperature
- NTC type is more commonly used.
- Can be quite sensitive, but not as accurate as other types of temperature sensors.
- Also used to limit starting current.

Thermocouple

- Involves junctions of two different metals.
- Generates a small voltage that is roughly proportional to the temperature difference of the two junctions.
- Based on Seebeck effect
- Many types available: Types K, J, N, R, S, B, T, E, and others.
- Advantage: Wide range (from -270 ℃ to 1700 ℃)
- Advantage: Requires no external power
- Disadvantage: Output is quite small, usually requires amplification
- Disadvantage: Amplification can be challenging

Resistance Thermometer

Resistance Temperature Detector (RTD)

- The resistivity of metals is a linear function of temperature over a wide range of temperatures.
- Usually a very pure form of the metal is used.
- A resistor made of the metal is enclosed in some form of protective housing.
- Commonly used metals: Platinum, Copper, Nickel
- Platinum can work till 600 ℃.

Temperature Coefficient of Resistance (TCR)

Let the resistance of a resistor be R_{ref} at temperature T_{ref} and R at temperature T. Then for metal resistors over a wide range of temperatures,

$$R = R_{\mathrm{ref}}[1 + \alpha(T - T_{\mathrm{ref}})] \tag{1}$$

where α is called the temperature coefficient of resistance (TCR).
T_{ref} is usually 20 ℃.

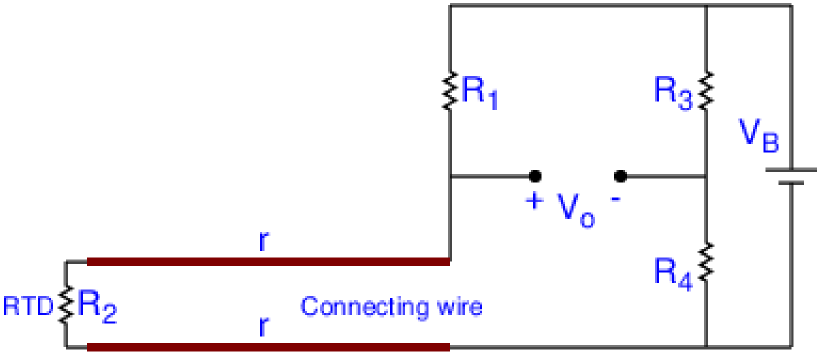
$$\alpha = \frac{R - R_{\mathrm{ref}}}{R_{\mathrm{ref}}(T - T_{\mathrm{ref}})} = \frac{\Delta R}{R_{\mathrm{ref}}\Delta T} \tag{2}$$

Units: per ℃

TCR of Commonly Used Metals

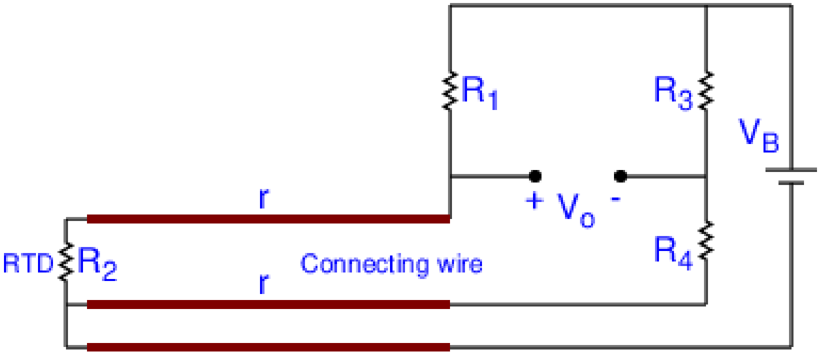
Metal	α
Platinum	3.925×10^{-3} °C^{-1}
Copper	3.9×10^{-3} °C^{-1}
Aluminium	3.9×10^{-3} °C^{-1}
Gold	3.4×10^{-3} °C^{-1}
Silver	3.8×10^{-3} °C^{-1}
Tungsten	4.5×10^{-3} °C^{-1}
Iron	5.0×10^{-3} °C^{-1}
Nickel	6.0×10^{-3} °C^{-1}
Tin	4.5×10^{-3} °C^{-1}
Lead	3.9×10^{-3} °C^{-1}

Two-wire Configuration



Disadvantage: r depends on the length of the connecting wire and only affects R_2.

Three-wire Configuration



Advantage: r affects both R_2 and R_4 equally.

Silicon Bandgap Temperature Sensor

The difference in voltage drop across two identical diodes or base to emitter junctions:

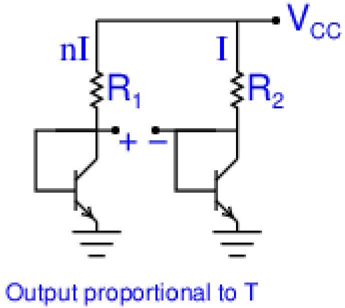
$$\Delta V_{BE} = \frac{kT}{q} \ln\left(\frac{I_{C1}}{I_{C2}}\right) \tag{3}$$

Here I_{C1} and I_{C2} are the diode or the collector currents.

- Can be part of an integrated circuit
- Reasonably accurate
- Inexpensive
- Example: LM35 temperature sensor IC

PTAT Temperature Sensor

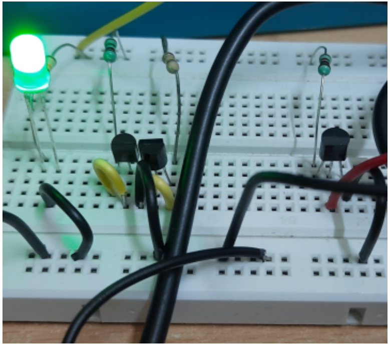
PTAT: Proportional to Absolute Temperature



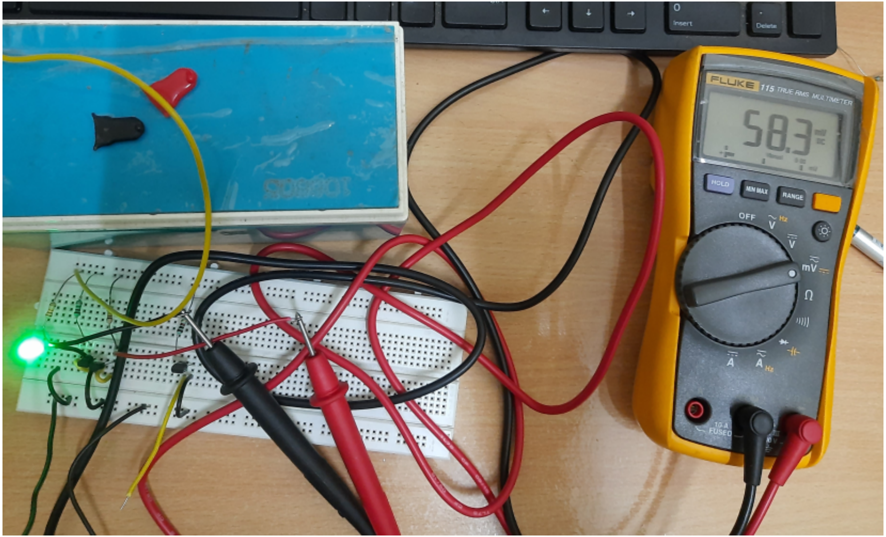
PTAT Circuit

$$\Delta V_{\mathrm{BE}} = \frac{kT}{q} \ln \left(\frac{I_{\mathrm{C1}}}{I_{\mathrm{C2}}} \right) = \frac{kT}{q} \ln n \qquad (4)$$

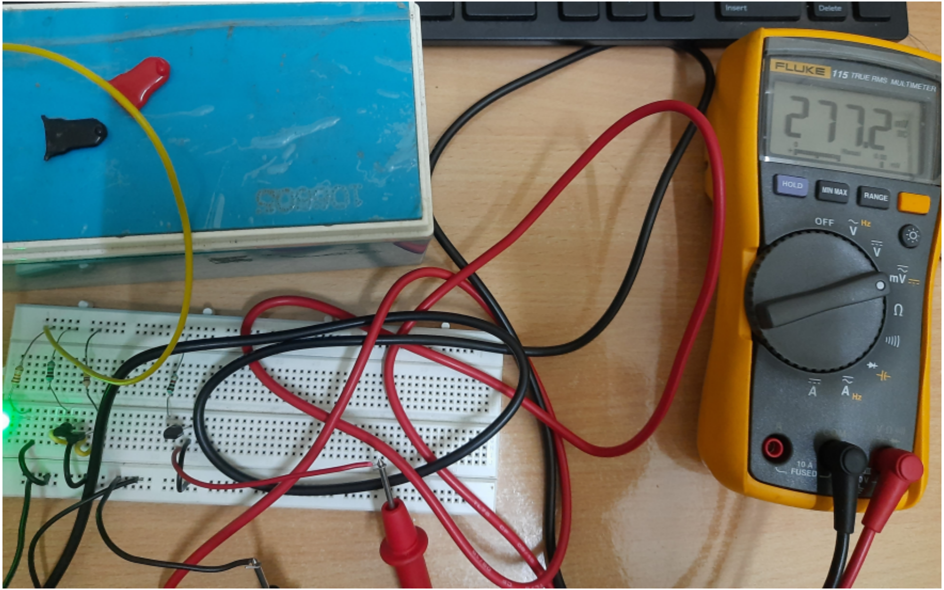
PTAT Circuit



PTAT Output


Insert
Delete
FLUKE
115 TRUE RMS MULTIMETER
58.3
HOLD
MIN MAX
RANGE
OFF
mV
10 A
FUSED

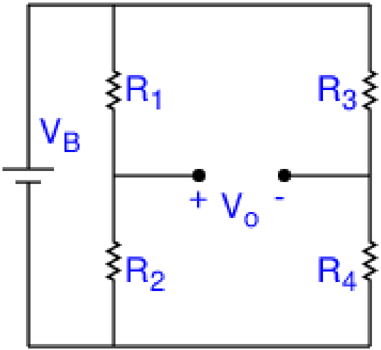
LM35 Output


FLUKE 115 TRUE RMS MULTIMETER
277.2
HOLD
MIN MAX
RANGE
OFF
10 A FUSED
Insert
Delete

Example PTAT Calculation

In the PTAT constructed, $\frac{I_{C1}}{I_{C2}} = n \approx 10$.
From the LM35 reading, T is 27.72 celsius or $27.72 + 273.15$ kelvin.
Calculation:
$n = 10$.
$T = 300.87\,\mathrm{K}$.
Boltzmann constant: $k = 1.380\,649 \times 10^{-23}\,\mathrm{J\,K^{-1}}$.
Elementary charge: $q = 1.602\,176\,634 \times 10^{-19}\,\mathrm{C}$.
$\frac{kT}{q} \ln n = 59.699\,\mathrm{mV}$ which is close to the $58.3\,\mathrm{mV}$ reading.

Practice Problem: Wheatstone Bridge



$$V_o = V_B\left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) \tag{5}$$

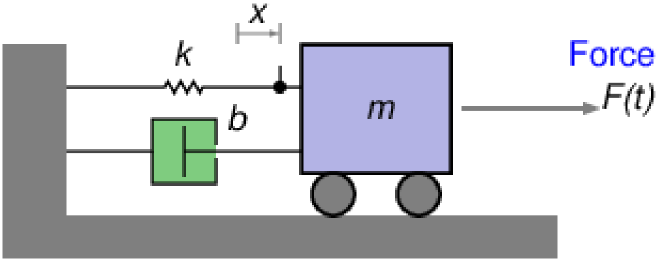
Practice Problems

1. The electrodes of a parallel plate capacitor are circular discs, each having a radius of 10 cm. If the electrodes are separated by an air gap of 1 mm, calculate the capacitance neglecting fringing fields.
2. A sine wave signal having peak voltage 20 V and frequency 10 MHz is applied across a 1 nF capacitor. Calculate the peak current in the capacitor.
3. A circular coil has 10 turns of wire with radius 10 cm. Calculate the magnitude of B on the axis of the coil at a distance 5 cm from the centre of the loop due to a 10 A current in the coil. Assume that there are no magnetic materials near the coil.
4. A resistor constructed using platinum wire has resistance 100 Ω at 20 ℃. What will be its resistance at 10 ℃?
5. In the Wheatstone bridge shown in the previous slide, $V_B = 10\,\text{V}$, $R_1 = R_4 = 100\,\Omega$, and $R_2 = R_3 = 96\,\Omega$. Calculate V_0.

Answers to Practice Problems

1. 278.157 pF
2. 1.256 64 A
3. 44.9588 µT
4. 96.075 Ω
5. −204.082 mV

Spring-Mass-Dashpot System: Modelling



x: Displacement of the mass from its equilibrium position

$$m\ddot{x} + b\dot{x} + kx = F \tag{6}$$

F: Force
$v = \dot{x}$: Velocity
Relationship between force and velocity:

$$m\ddot{v} + b\dot{v} + kv = \dot{F} \tag{7}$$

Tension in the Dashpot

- Here the applied force $F(t)$ is the input.
- We could consider the velocity $v(t)$ as the output.
- A better choice is to consider the tension in the dashpot, $F_d(t) = bv(t)$, as the output.
- $F_d(t)$ is the force endured by the dashpot.
- Having both input and output as forces makes the mathematics neater.

Relationship between $F(t)$ and $F_d(t)$:

$$m\ddot{F}_d + b\dot{F}_d + kF_d = b\dot{F} \tag{8}$$

Transfer Function

The transfer function is

$$T(s) = \frac{\mathcal{F}_d(s)}{\mathcal{F}(s)} = \frac{bs}{ms^2 + bs + k} = \frac{(b/m)s}{s^2 + (b/m)s + k/m} \tag{9}$$

Or,

$$T(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2},$$

where,

$$\omega_0 = \sqrt{\frac{k}{m}},$$

and

$$b/m = 2\alpha.$$

Terminology

ω_0 is the angular frequency of oscillations in the absence of damping.
α is called the decay constant.
Both ω_0 and α have dimensions of the inverse of time.
Alternate Notation: ω_n for ω_0, $2\zeta\omega_n$ for 2α
See for example, Section 3.5 of *Linear Control System Analysis and Design with MATLAB* by D'Azzo, Houpis and Sheldon.

Interpretation

An input of e^{st} produces an output of $T(s)e^{st}$ in the steady state, that is after the transients have died down.

In this case, the transients will decay to zero because both the roots of $s^2 + 2\alpha s + \omega_0^2 = 0$ have negative real parts.

Sinusoidal Input

Let $T(j\omega) = U + jV$, so that $T(-j\omega) = U - jV$.
Input $e^{j\omega t}$ produces output $T(j\omega)e^{j\omega t} =$
$U\cos(\omega t) - V\sin(\omega t) + j\,[U\sin(\omega t) + V\cos(\omega t)]$.
Input $e^{-j\omega t}$ produces output $T(-j\omega)e^{-j\omega t} =$
$U\cos(\omega t) - V\sin(\omega t) - j\,[U\sin(\omega t) + V\cos(\omega t)]$.
Input $\cos(\omega t)$ produces output $U\cos(\omega t) - V\sin(\omega t)$, which is same as

$$\sqrt{U^2 + V^2}\left(\frac{U}{\sqrt{U^2 + V^2}}\cos(\omega t) - \frac{V}{\sqrt{U^2 + V^2}}\sin(\omega t)\right)$$

$$= \sqrt{U^2 + V^2}\cos(\omega t + \Phi) = |T(j\omega)|\cos(\omega t + \Phi)$$

where $\Phi = \arctan(V/U)$, more correctly `atan2(V,U)`, is the angle of $T(j\omega)$.

Meaning of $T(j\omega)$

So for sinusoidal input, the output is also sinusoidal, the amplitude being multiplied by $|T(j\omega)|$, the magnitude of $T(j\omega)$, and the phase being shifted by the angle of $T(j\omega)$.

$|T(j\omega)|$

Here

$$|T(j\omega)| = \left|\frac{2\alpha j\omega}{2\alpha j\omega + \omega_0^2 - \omega^2}\right| = \frac{1}{\sqrt{1 + \left(\frac{\omega_0^2 - \omega^2}{2\alpha\omega}\right)^2}}. \tag{10}$$

Maximum Output: $|T(j\omega)| = 1$ when $\omega = \pm\omega_0$.
ω_0 is called the centre angular frequency.

Sharpness of Response

Half-power Output: This happens when $|T(j\omega)| = 1/\sqrt{2}$.
Or,

$$\frac{\omega_0^2 - \omega^2}{2\alpha\omega} = \pm 1 \tag{11}$$

The two quadratic equations to be solved are

$$\omega^2 - 2\alpha\omega - \omega_0^2 = 0, \tag{12}$$

and

$$\omega^2 + 2\alpha\omega - \omega_0^2 = 0. \tag{13}$$

Half-power Angular Frequencies

The positive root of Eq. 12, called the *upper half-power angular frequency* is

$$\omega_+ = \alpha + \sqrt{\omega_0^2 + \alpha^2} \tag{14}$$

The positive root of Eq. 13, called the *lower half-power angular frequency* is

$$\omega_- = -\alpha + \sqrt{\omega_0^2 + \alpha^2} \tag{15}$$

Note: The negative root of Eq. 12 is $-\omega_-$, and the negative root of Eq. 13 is $-\omega_+$.
$\Delta\omega = \omega_+ - \omega_- = 2\alpha$ is called the half-power bandwidth.
Note that

$$\omega_+ \omega_- = \omega_0^2. \tag{16}$$

Quality Factor Q

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{2\alpha} \tag{17}$$

is a measure of the selectivity or the sharpness of response. A higher Q makes the response more selective.
So

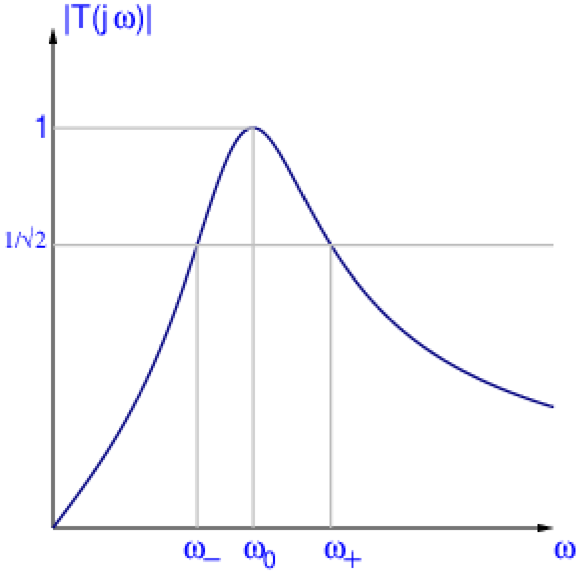
$$2\alpha = \frac{\omega_0}{Q}. \tag{18}$$

In view of this,

$$T(s) = \frac{\frac{\omega_0 s}{Q}}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}.$$

Whenever we see a quadratic denominator, we use the Q notation, even if the system is *not* a bandpass system.

Half-power Angular Frequencies Shown for $Q = 1.5$



ω_+ and ω_- in terms of ω_0 and Q

$$\omega_+ = \omega_0 \left(\sqrt{1 + \frac{1}{4Q^2}} + \frac{1}{2Q} \right). \tag{19}$$

$$\omega_- = \omega_0 \left(\sqrt{1 + \frac{1}{4Q^2}} - \frac{1}{2Q} \right). \tag{20}$$

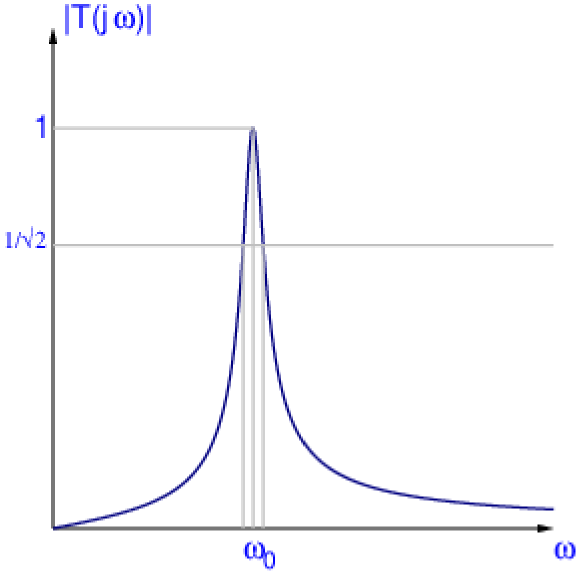
Also, remember that $\omega_+\omega_- = \omega_0^2$, and $\omega_+ - \omega_- = \omega_0/Q$.
Note that,

$$\frac{\omega_+}{\omega_0} - \frac{\omega_0}{\omega_+} = \frac{1}{Q}, \tag{21}$$

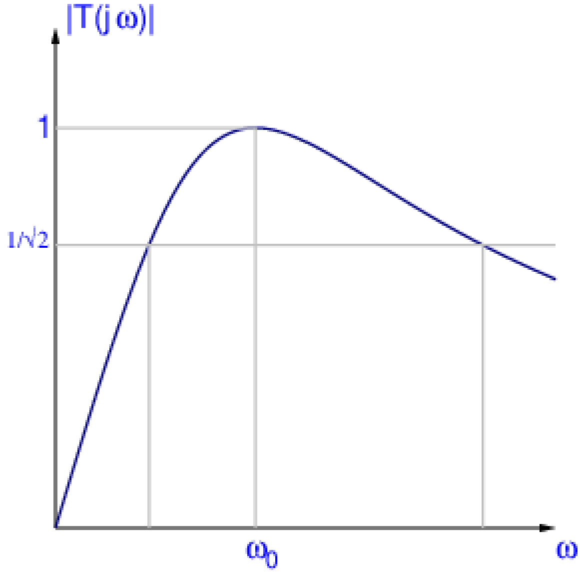
and

$$\frac{\omega_-}{\omega_0} - \frac{\omega_0}{\omega_-} = -\frac{1}{Q}. \tag{22}$$

$|T(j\omega)|$ for $Q = 10$



$|T(j\omega)|$ for $Q = 0.6$



BPF Phase

$$T(j\omega) = \frac{2\alpha j\omega}{2\alpha j\omega + \omega_0^2 - \omega^2} = \frac{j\omega\omega_0/Q}{j\omega\omega_0/Q + \omega_0^2 - \omega^2} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}. \qquad (23)$$

Phase angle is

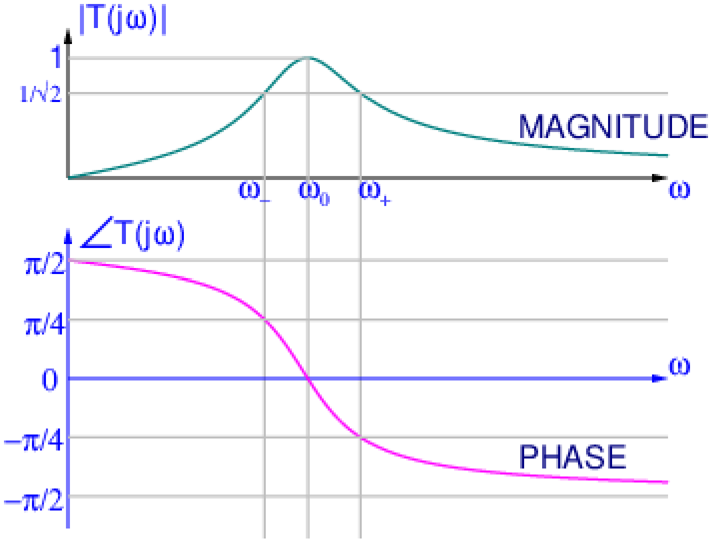
$$\underline{/T(j\omega)} = \arctan\left(Q\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)\right). \qquad (24)$$

Special values:

- $\underline{/T(j0)} = \pi/2$.
- $\underline{/T(j\omega_0)} = 0$.
- $\underline{/T(j\infty)} = -\pi/2$.
- $\underline{/T(j\omega_-)} = \pi/4$.
- $\underline{/T(j\omega_+)} = -\pi/4$.

Phase is important because it is often easier to measure.

BPF magnitude and phase on the same plot



BPF MAGNITUDE AND PHASE PLOTS FOR Q = 2.5

Second-order BPF: More general form

We studied a transfer function of the form

$$T(s) = \frac{\frac{\omega_0 s}{Q}}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}$$

that occurs in many applications.
The meanings of the Q and ω_0 parameters were understood.
A slightly more general form for the second-order BPF transfer function is

$$T(s) = \frac{H\frac{\omega_0 s}{Q}}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}.$$

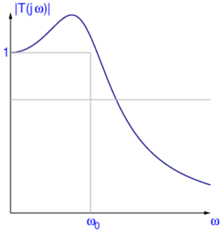
Here H is constant gain or loss factor, useful in systems with amplification or extra losses.

LPF System

Example: MEMS Accelerometer
Input is applied force, output can be the displacement x.
Or, to simplify the mathematics, let the force in the spring, kx, be the output. Then

$$T(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}. \tag{25}$$

Even here, the symbol Q is used.



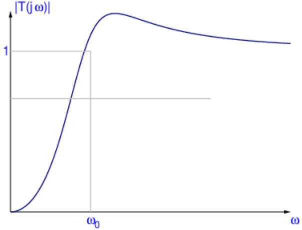
LPF $|T(j\omega)|$ shown for $Q = 1.1$.

HPF System

Example: MEMS Accelerometer
Input is applied force, output can be the acceleration $\ddot{x}$.
Or, to simplify the mathematics, let the force acting on the mass, $m\ddot{x}$, be the output. Then

$$T(s) = \frac{s^2}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}. \tag{26}$$

The same symbol Q is used.



LPF $|T(j\omega)|$ shown for $Q = 1.1$.