IN 221 (AUG) 3:0
Sensors and Transducers
Electromagnetic Sensors and Transducers
Lecture 4

A. Mohanty

Department of Instrumentation and Applied Physics (IAP)
Indian Institute of Science
Bangalore 560012

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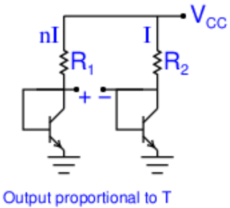
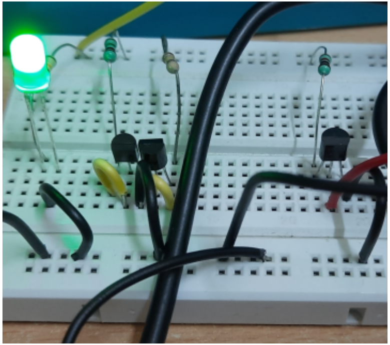
PTAT Temperature Sensor



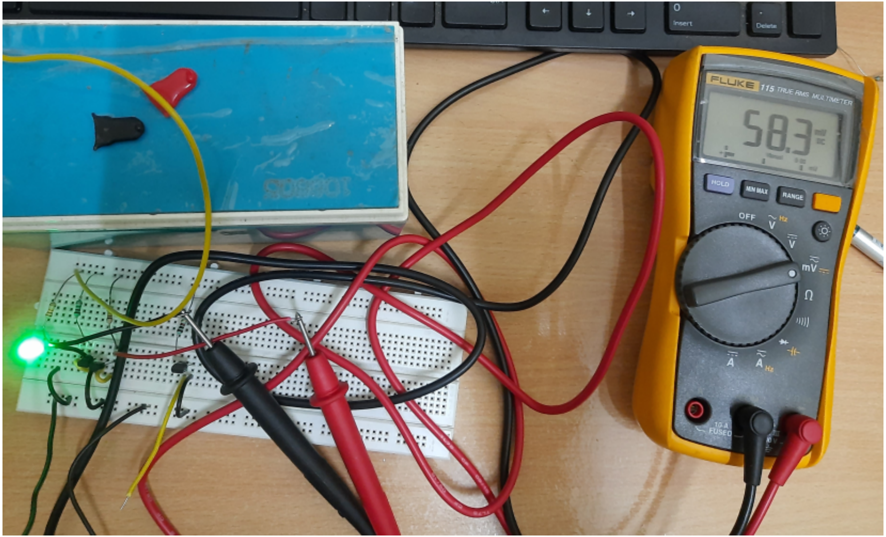
PTAT Circuit

$$\Delta V_{\mathrm{BE}} = \frac{kT}{q} \ln \left(\frac{I_{\mathrm{C1}}}{I_{\mathrm{C2}}} \right) \quad (1)$$

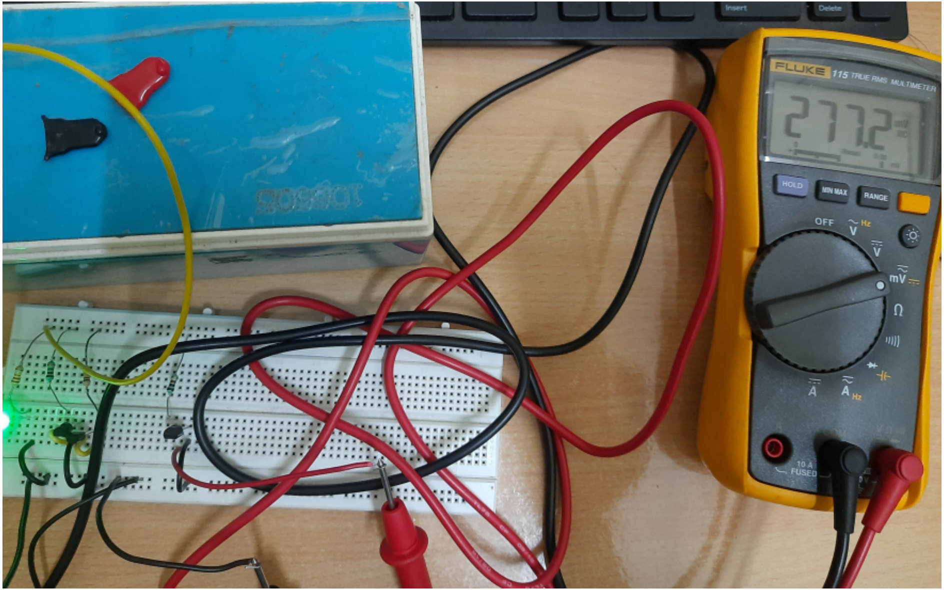
PTAT Circuit



PTAT Output


FLUKE
115 TRUE RMS MULTIMETER
58.3
HOLD
MIN MAX
RANGE
OFF
Insert
Delete

LM35 Output



Insert
Delete
FLUKE
115 TRUE RMS MULTIMETER
HOLD
MIN MAX
RANGE
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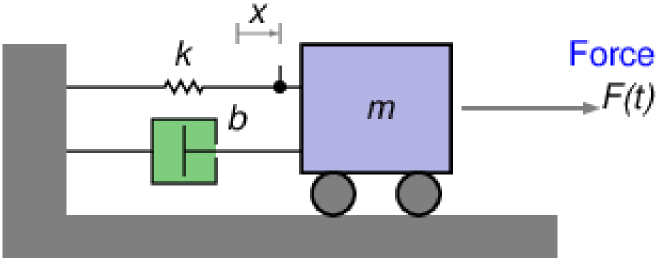
10 A FUSED

Spring-Mass-Dashpot System: Modelling



x: Displacement of the mass from its equilibrium position

$$m\ddot{x} + b\dot{x} + kx = F \tag{2}$$

F: Force
$v = \dot{x}$: Velocity
Relationship between force and velocity:

$$m\ddot{v} + b\dot{v} + kv = \dot{F} \tag{3}$$

Tension in the Dashpot

- Here the applied force $F(t)$ is the input.
- We could consider the velocity $v(t)$ as the output.
- A better choice is to consider the tension in the dashpot, $F_d(t) = bv(t)$, as the output.
- $F_d(t)$ is the force endured by the dashpot.
- Having both input and output as forces makes the mathematics neater.

Relationship between $F(t)$ and $F_d(t)$:

$$m\ddot{F}_d + b\dot{F}_d + kF_d = b\dot{F} \tag{4}$$

Transfer Function

The transfer function is

$$T(s) = \frac{\mathcal{F}_d(s)}{\mathcal{F}(s)} = \frac{bs}{ms^2 + bs + k} = \frac{(b/m)s}{s^2 + (b/m)s + k/m} \tag{5}$$

Or,

$$T(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2},$$

where,

$$\omega_0 = \sqrt{\frac{k}{m}},$$

and

$$b/m = 2\alpha.$$

Terminology

ω_0 is the angular frequency of oscillations in the absence of damping.
α is called the decay constant.
Both ω_0 and α have dimensions of the inverse of time.
Alternate Notation: ω_n for ω_0, $2\zeta\omega_n$ for 2α
See for example, Section 3.5 of *Linear Control System Analysis and Design with MATLAB* by D'Azzo, Houpis and Sheldon.

Interpretation

An input of e^{st} produces an output of $T(s)e^{st}$ in the steady state, that is after the transients have died down.

In this case, the transients will decay to zero because both the roots of $s^2 + 2\alpha s + \omega_0^2 = 0$ have negative real parts.

Sinusoidal Input

Let $T(j\omega) = U + jV$, so that $T(-j\omega) = U - jV$.
Input $e^{j\omega t}$ produces output $T(j\omega)e^{j\omega t} =$
$U\cos(\omega t) - V\sin(\omega t) + j\left[U\sin(\omega t) + V\cos(\omega t)\right]$.
Input $e^{-j\omega t}$ produces output $T(-j\omega)e^{-j\omega t} =$
$U\cos(\omega t) - V\sin(\omega t) - j\left[U\sin(\omega t) + V\cos(\omega t)\right]$.
Input $\cos(\omega t)$ produces output $U\cos(\omega t) - V\sin(\omega t)$, which is same as

$$\sqrt{U^2+V^2}\left(\frac{U}{\sqrt{U^2+V^2}}\cos(\omega t) - \frac{V}{\sqrt{U^2+V^2}}\sin(\omega t)\right)$$

$$= \sqrt{U^2+V^2}\cos(\omega t + \Phi) = |T(j\omega)|\cos(\omega t + \Phi)$$

where $\Phi = \arctan(V/U)$, more correctly `atan2(V,U)`, is the angle of $T(j\omega)$.

Meaning of $T(j\omega)$

So for sinusoidal input, the output is also sinusoidal, the amplitude being multiplied by $|T(j\omega)|$, the magnitude of $T(j\omega)$, and the phase being shifted by the angle of $T(j\omega)$.

Here

$$|T(j\omega)| = \left| \frac{2\alpha j\omega}{2\alpha j\omega + \omega_0^2 - \omega^2} \right| = \frac{1}{\sqrt{1 + \left(\frac{\omega_0^2 - \omega^2}{2\alpha\omega} \right)^2}}. \tag{6}$$

Maximum Output: $|T(j\omega)| = 1$ when $\omega = \pm\omega_0$.
ω_0 is called the centre angular frequency.

Sharpness of Response

Half-power Output: This happens when $|T(j\omega)| = 1/\sqrt{2}$.
Or,

$$\frac{\omega_0^2 - \omega^2}{2\alpha\omega} = \pm 1 \tag{7}$$

The two quadratic equations to be solved are

$$\omega^2 - 2\alpha\omega - \omega_0^2 = 0, \tag{8}$$

and

$$\omega^2 + 2\alpha\omega - \omega_0^2 = 0. \tag{9}$$

Half-power Angular Frequencies

The positive root of Eq. 8, called the *upper half-power angular frequency* is

$$\omega_+ = \alpha + \sqrt{\omega_0^2 + \alpha^2} \tag{10}$$

The positive root of Eq. 9, called the *lower half-power angular frequency* is

$$\omega_- = -\alpha + \sqrt{\omega_0^2 + \alpha^2} \tag{11}$$

Note: The negative root of Eq. 8 is $-\omega_-$, and the negative root of Eq. 9 is $-\omega_+$.
$\Delta\omega = \omega_+ - \omega_- = 2\alpha$ is called the half-power bandwidth.
Note that

$$\omega_+\omega_- = \omega_0^2. \tag{12}$$

Quality Factor Q

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{2\alpha} \tag{13}$$

is a measure of the selectivity or the sharpness of response. A higher Q makes the response more selective.
So

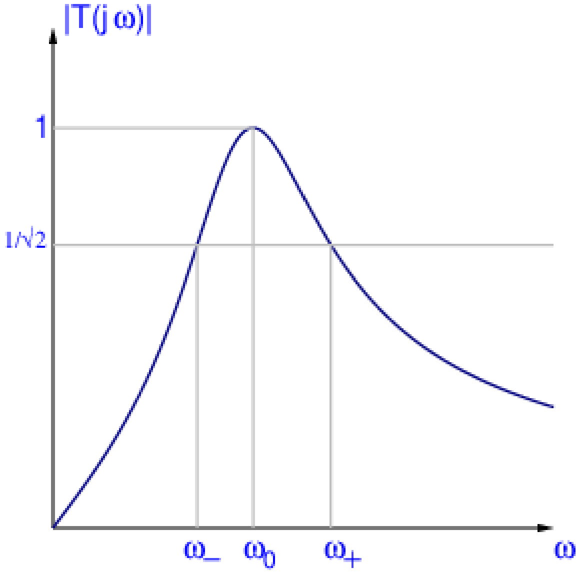
$$2\alpha = \frac{\omega_0}{Q}. \tag{14}$$

In view of this,

$$T(s) = \frac{\frac{\omega_0 s}{Q}}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}.$$

Whenever we see a quadratic denominator, we use the Q notation, even if the system is *not* a bandpass system.

Half-power Angular Frequencies Shown for $Q = 1.5$



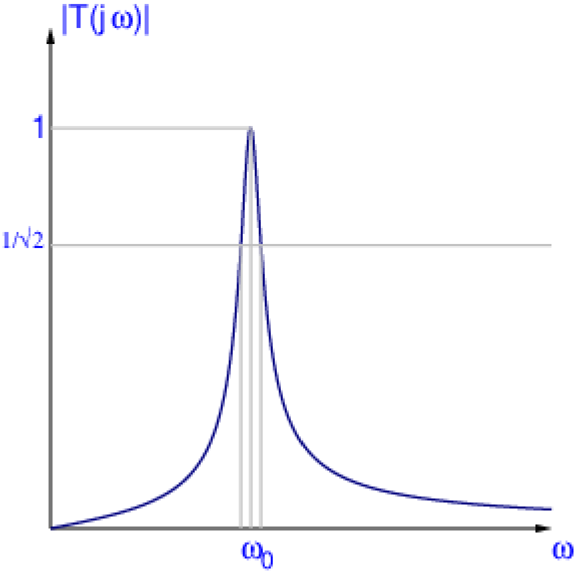
ω_+ and ω_- in terms of ω_0 and Q

$$\omega_+ = \omega_0 \left(\sqrt{1 + \frac{1}{4Q^2}} + \frac{1}{2Q} \right). \tag{15}$$

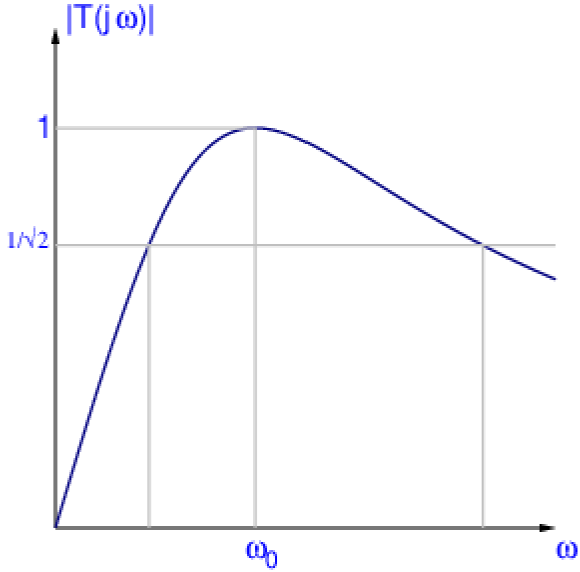
$$\omega_- = \omega_0 \left(\sqrt{1 + \frac{1}{4Q^2}} - \frac{1}{2Q} \right). \tag{16}$$

Also, remember that $\omega_+ \omega_- = \omega_0^2$, and $\omega_+ - \omega_- = \omega_0 / Q$.

$|T(j\omega)|$ for $Q = 10$



$|T(j\omega)|$ for $Q = 0.6$

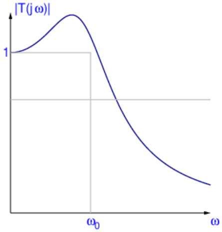


LPF System

Example: MEMS Accelerometer
Input is applied force, output can be the displacement x.
Or, to simplify the mathematics, let the force in the spring, kx, be the output. Then

$$T(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}. \tag{17}$$

Even here, the symbol Q is used.

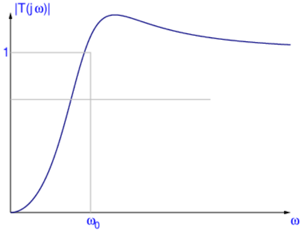


LPF $|T(j\omega)|$ shown for $Q = 1.1$.

Example: MEMS Accelerometer
Input is applied force, output can be the acceleration $\ddot{x}$.
Or, to simplify the mathematics, let the force acting on the mass, $m\ddot{x}$, be the output. Then

$$T(s) = \frac{s^2}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}. \tag{18}$$

The same symbol Q is used.



LPF $|T(j\omega)|$ shown for $Q = 1.1$.