IN 221 (AUG) 3:0
Sensors and Transducers
Electromagnetic Sensors and Transducers
Lecture 5

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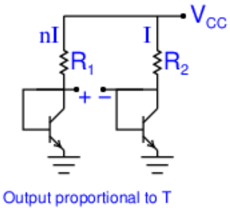
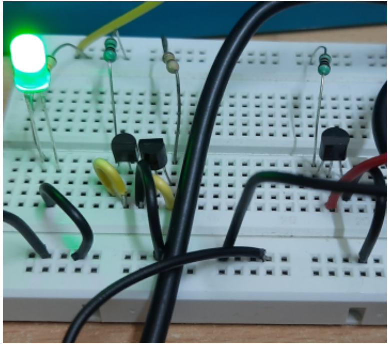
PTAT Temperature Sensor



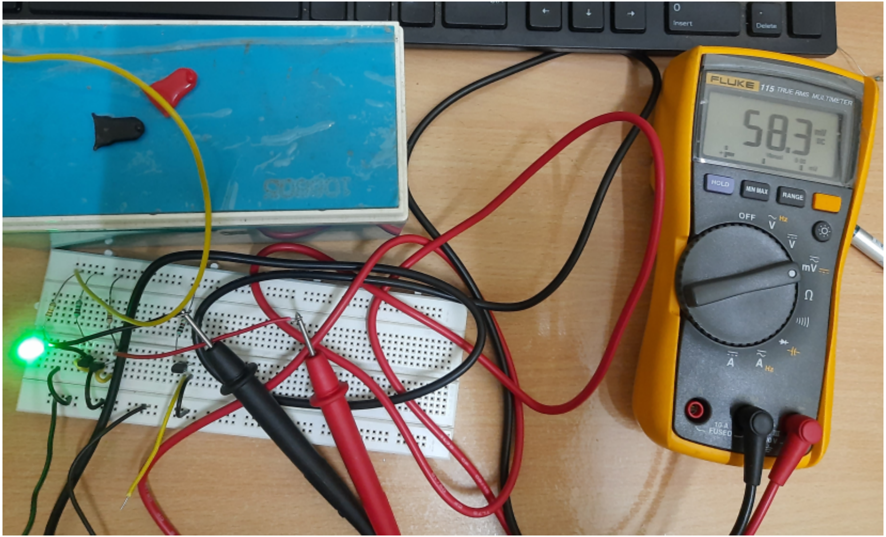
PTAT Circuit

$$\Delta V_{\mathrm{BE}} = \frac{kT}{q} \ln \left(\frac{I_{\mathrm{C1}}}{I_{\mathrm{C2}}} \right) \tag{1}$$

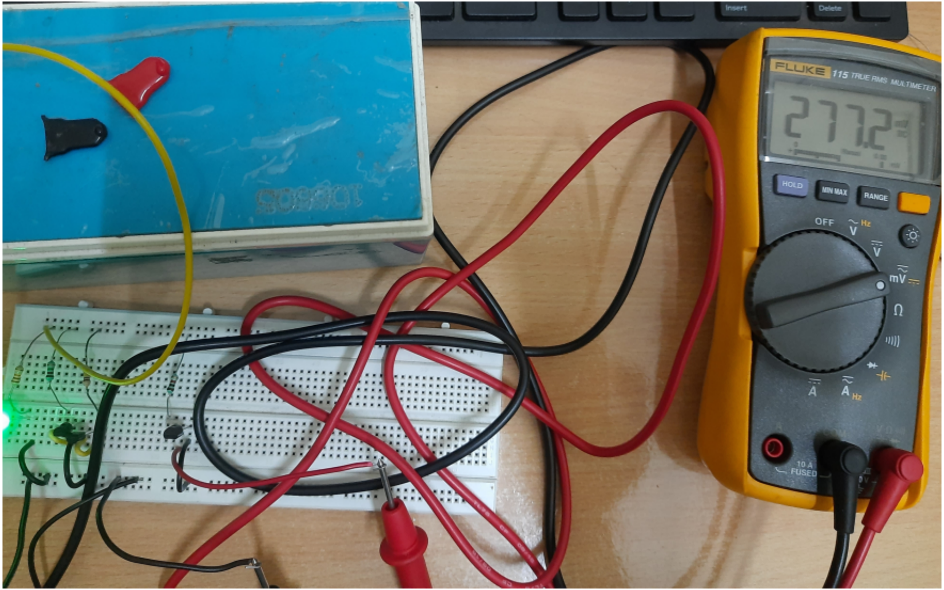
PTAT Circuit



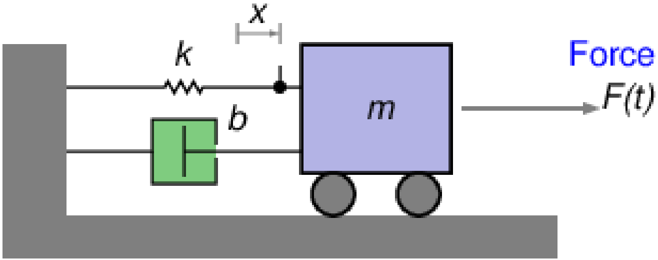
PTAT Output


FLUKE
115 TRUE RMS MULTIMETER
58.3
HOLD
MIN MAX
RANGE
OFF
Insert
Delete

LM35 Output


FLUKE
115 TRUE RMS MULTIMETER
277.2
HOLD
MIN MAX
RANGE
OFF
Insert
Delete

Spring-Mass-Dashpot System: Modelling



x: Displacement of the mass from its equilibrium position

$$m\ddot{x} + b\dot{x} + kx = F \tag{2}$$

F: Force
$v = \dot{x}$: Velocity
Relationship between force and velocity:

$$m\ddot{v} + b\dot{v} + kv = \dot{F} \tag{3}$$

Tension in the Dashpot

- Here the applied force $F(t)$ is the input.
- We could consider the velocity $v(t)$ as the output.
- A better choice is to consider the tension in the dashpot, $F_d(t) = bv(t)$, as the output.
- $F_d(t)$ is the force endured by the dashpot.
- Having both input and output as forces makes the mathematics neater.

Relationship between $F(t)$ and $F_d(t)$:

$$m\ddot{F}_d + b\dot{F}_d + kF_d = b\dot{F} \tag{4}$$

Transfer Function

The transfer function is

$$T(s) = \frac{\mathcal{F}_d(s)}{\mathcal{F}(s)} = \frac{bs}{ms^2 + bs + k} = \frac{(b/m)s}{s^2 + (b/m)s + k/m} \tag{5}$$

Or,

$$T(s) = \frac{2\alpha s}{s^2 + 2\alpha s + \omega_0^2},$$

where,

$$\omega_0 = \sqrt{\frac{k}{m}},$$

and

$$b/m = 2\alpha.$$

Terminology

ω_0 is the angular frequency of oscillations in the absence of damping.
α is called the decay constant.
Both ω_0 and α have dimensions of the inverse of time.
Alternate Notation: ω_n for ω_0, $2\zeta\omega_n$ for 2α
See for example, Section 3.5 of *Linear Control System Analysis and Design with MATLAB* by D'Azzo, Houpis and Sheldon.

Interpretation

An input of e^{st} produces an output of $T(s)e^{st}$ in the steady state, that is after the transients have died down.

In this case, the transients will decay to zero because both the roots of $s^2 + 2\alpha s + \omega_0^2 = 0$ have negative real parts.

Sinusoidal Input

Let $T(j\omega) = U + jV$, so that $T(-j\omega) = U - jV$.
Input $e^{j\omega t}$ produces output $T(j\omega)e^{j\omega t} =$
$U\cos(\omega t) - V\sin(\omega t) + j\,[U\sin(\omega t) + V\cos(\omega t)]$.
Input $e^{-j\omega t}$ produces output $T(-j\omega)e^{-j\omega t} =$
$U\cos(\omega t) - V\sin(\omega t) - j\,[U\sin(\omega t) + V\cos(\omega t)]$.
Input $\cos(\omega t)$ produces output $U\cos(\omega t) - V\sin(\omega t)$, which is same as

$$\sqrt{U^2+V^2}\left(\frac{U}{\sqrt{U^2+V^2}}\cos(\omega t) - \frac{V}{\sqrt{U^2+V^2}}\sin(\omega t)\right)$$

$$= \sqrt{U^2+V^2}\cos(\omega t + \Phi) = |T(j\omega)|\cos(\omega t + \Phi)$$

where $\Phi = \arctan(V/U)$, more correctly `atan2(V,U)`, is the angle of $T(j\omega)$.

Meaning of $T(j\omega)$

So for sinusoidal input, the output is also sinusoidal, the amplitude being multiplied by $|T(j\omega)|$, the magnitude of $T(j\omega)$, and the phase being shifted by the angle of $T(j\omega)$.

$|T(j\omega)|$

Here

$$|T(j\omega)| = \left|\frac{2\alpha j\omega}{2\alpha j\omega + \omega_0^2 - \omega^2}\right| = \frac{1}{\sqrt{1 + \left(\frac{\omega_0^2 - \omega^2}{2\alpha\omega}\right)^2}}. \tag{6}$$

Maximum Output: $|T(j\omega)| = 1$ when $\omega = \pm\omega_0$.
ω_0 is called the centre angular frequency.

Sharpness of Response

Half-power Output: This happens when $|T(j\omega)| = 1/\sqrt{2}$.
Or,

$$\frac{\omega_0^2 - \omega^2}{2\alpha\omega} = \pm 1 \tag{7}$$

The two quadratic equations to be solved are

$$\omega^2 - 2\alpha\omega - \omega_0^2 = 0, \tag{8}$$

and

$$\omega^2 + 2\alpha\omega - \omega_0^2 = 0. \tag{9}$$

Half-power Angular Frequencies

The positive root of Eq. 8, called the *upper half-power angular frequency* is

$$\omega_+ = \alpha + \sqrt{\omega_0^2 + \alpha^2} \tag{10}$$

The positive root of Eq. 9, called the *lower half-power angular frequency* is

$$\omega_- = -\alpha + \sqrt{\omega_0^2 + \alpha^2} \tag{11}$$

Note: The negative root of Eq. 8 is $-\omega_-$, and the negative root of Eq. 9 is $-\omega_+$.
$\Delta\omega = \omega_+ - \omega_- = 2\alpha$ is called the half-power bandwidth.
Note that

$$\omega_+\omega_- = \omega_0^2. \tag{12}$$

Quality Factor Q

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{\omega_0}{2\alpha} \tag{13}$$

is a measure of the selectivity or the sharpness of response. A higher Q makes the response more selective.
So

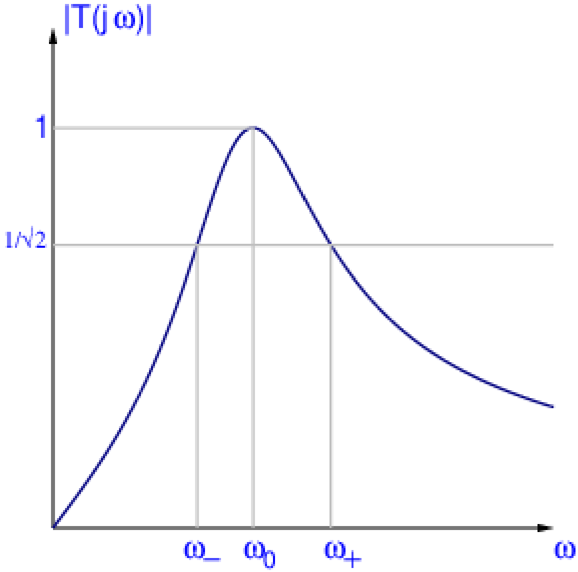
$$2\alpha = \frac{\omega_0}{Q}. \tag{14}$$

In view of this,

$$T(s) = \frac{\frac{\omega_0 s}{Q}}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}.$$

Whenever we see a quadratic denominator, we use the Q notation, even if the system is *not* a bandpass system.

Half-power Angular Frequencies Shown for $Q = 1.5$



ω_+ and ω_- in terms of ω_0 and Q

$$\omega_+ = \omega_0 \left(\sqrt{1 + \frac{1}{4Q^2}} + \frac{1}{2Q} \right). \tag{15}$$

$$\omega_- = \omega_0 \left(\sqrt{1 + \frac{1}{4Q^2}} - \frac{1}{2Q} \right). \tag{16}$$

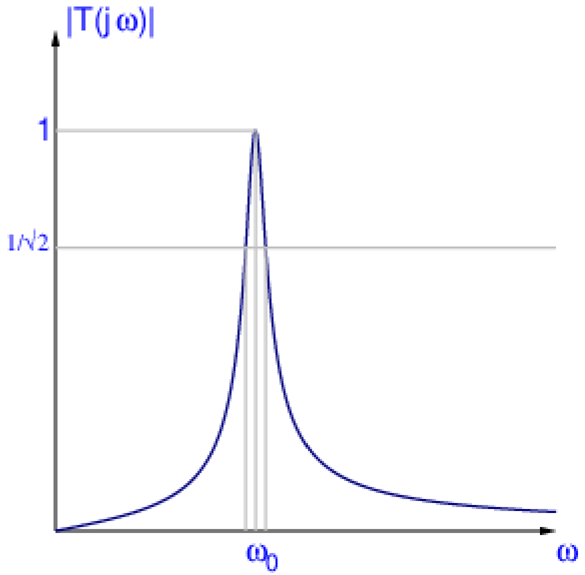
Also, remember that $\omega_+\omega_- = \omega_0^2$, and $\omega_+ - \omega_- = \omega_0/Q$.
Note that,

$$\frac{\omega_+}{\omega_0} - \frac{\omega_0}{\omega_+} = \frac{1}{Q}, \tag{17}$$

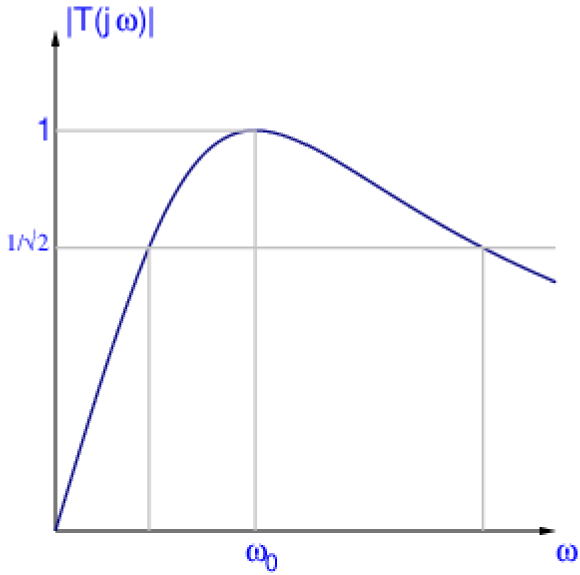
and

$$\frac{\omega_-}{\omega_0} - \frac{\omega_0}{\omega_-} = -\frac{1}{Q}. \tag{18}$$

$|T(j\omega)|$ for $Q = 10$



$|T(j\omega)|$ for $Q = 0.6$



BPF Phase

$$T(j\omega) = \frac{2\alpha j\omega}{2\alpha j\omega + \omega_0^2 - \omega^2} = \frac{j\omega\omega_0/Q}{j\omega\omega_0/Q + \omega_0^2 - \omega^2} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}. \tag{19}$$

Phase angle is

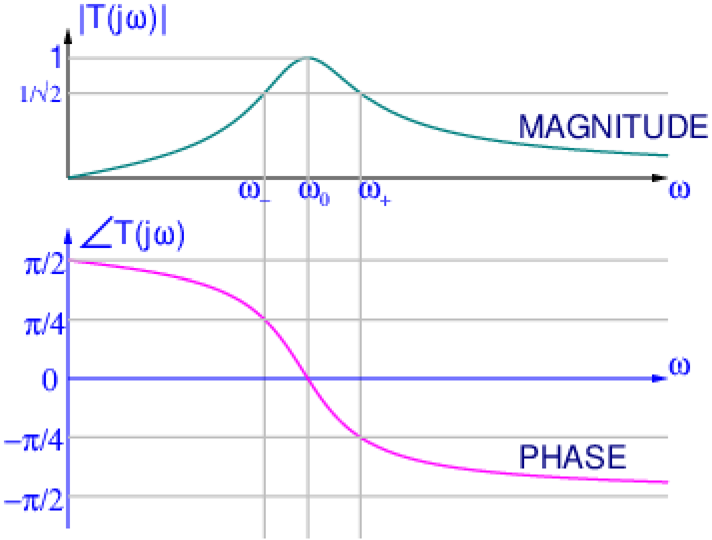
$$\underline{/T(j\omega)} = \arctan\left(Q\left(\frac{\omega_0}{\omega} - \frac{\omega}{\omega_0}\right)\right). \tag{20}$$

Special values:

- $\underline{/T(j0)} = \pi/2$.
- $\underline{/T(j\omega_0)} = 0$.
- $\underline{/T(j\infty)} = -\pi/2$.
- $\underline{/T(j\omega_-)} = \pi/4$.
- $\underline{/T(j\omega_+)} = -\pi/4$.

Phase is important because it is often easier to measure.

BPF magnitude and phase on the same plot



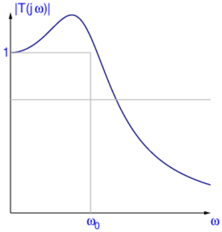
BPF MAGNITUDE AND PHASE PLOTS FOR Q = 2.5

LPF System

Example: MEMS Accelerometer
Input is applied force, output can be the displacement x.
Or, to simplify the mathematics, let the force in the spring, kx, be the output. Then

$$T(s) = \frac{\omega_0^2}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}. \tag{21}$$

Even here, the symbol Q is used.



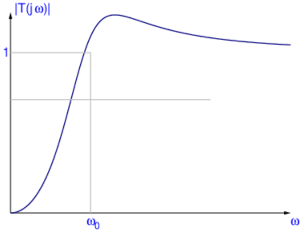
LPF $|T(j\omega)|$ shown for $Q = 1.1$.

HPF System

Example: MEMS Accelerometer
Input is applied force, output can be the acceleration $\ddot{x}$.
Or, to simplify the mathematics, let the force acting on the mass, $m\ddot{x}$, be the output. Then

$$T(s) = \frac{s^2}{s^2 + \frac{\omega_0 s}{Q} + \omega_0^2}. \tag{22}$$

The same symbol Q is used.



HPF $|T(j\omega)|$ shown for $Q = 1.1$.

Second Order LPF Magnitude Response

$$T(j\omega) = \frac{\omega_0^2}{\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}}$$

At what frequency is $|T(j\omega)|$ maximum?
The numerator is constant. The square of the magnitude of the denominator is

$$(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2} = \omega_0^4 + \omega^4 - 2\omega_0^2\omega^2 + \frac{\omega^2\omega_0^2}{Q^2}$$

$$= \omega_0^4 + \omega^4 - 2\omega_0^2\omega^2\left(1 - \frac{1}{2Q^2}\right)$$

We will try to complete squares here. The result depends on the value of Q.

Magnitude Response (continued)

If $Q \leq 1/\sqrt{2}$, all terms are non-negative and the denominator is an increasing function of ω.
In that case, $|T(j\omega)|$ has a maximum value of 1 at $\omega = 0$. For any other ω, $|T(j\omega)|$ is a monotonically decreasing function of $|\omega|$. We then say that there is *no peaking*.

Magnitude Response (continued)

If $Q > 1/\sqrt{2}$, we can complete the square to get the denominator magnitude squared as

$$\left(\omega^2 - \omega_0^2\left(1 - \frac{1}{2Q^2}\right)\right)^2 + \omega_0^4\frac{1}{Q^2}\left(1 - \frac{1}{4Q^2}\right)$$

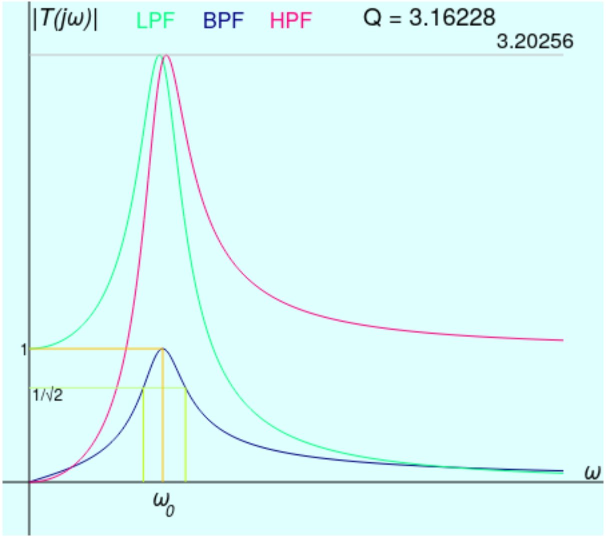
So $|T(j\omega)|$ is maximum when

$$|\omega| = \omega_L = \omega_0\sqrt{1 - \frac{1}{2Q^2}}. \tag{23}$$

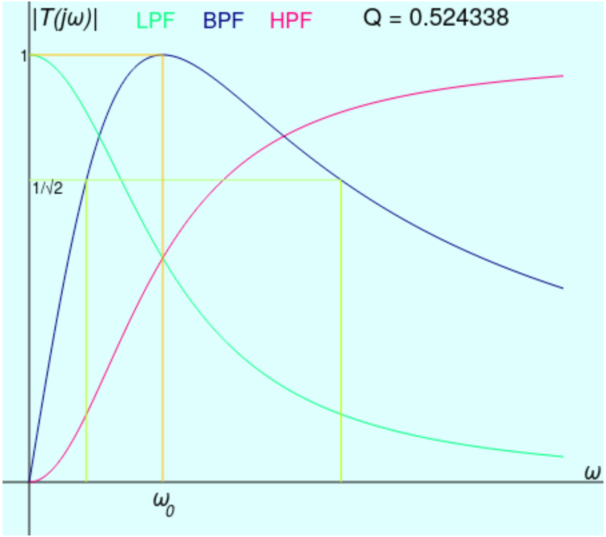
$$|T(j\omega_L)| = \frac{Q}{\sqrt{1 - \frac{1}{4Q^2}}} \tag{24}$$

This gives rise to *peaking*.
Also, note that $|T(j\omega_0)| = Q$, for the LPF.

Case of *Peaking*



Case of *No Peaking*



Pressure Sensors for Vacuum Systems

- Vacuum System: Uses pumps to remove air from a chamber.
- Very low pressures are needed.
- Some experiments cannot be done unless the pressure is lower than a specified value.
- Sensors which indicate the pressure accurately are needed.

Commonly used Pressure Units

- Pascal (Pa): 1 newton per square metre (SI Unit)
- 1 bar = 10^5 Pa = 100 kPa
- Technical Atmosphere: 1 at = 1 kgf per centimetre squared = 98066.5 Pa
- Standard Atmosphere: 101325 Pa = 760 Torr
- 1 Torr = 133.3224 Pa (approximately 1 mmHg) (named after Torricelli)
- 1 pound force per square inch = 1 lbf $/(\mathrm{in})^2$ = 6894.757 Pa

Low Pressure Sensors

Low pressure: Pressure less than 1 mbar

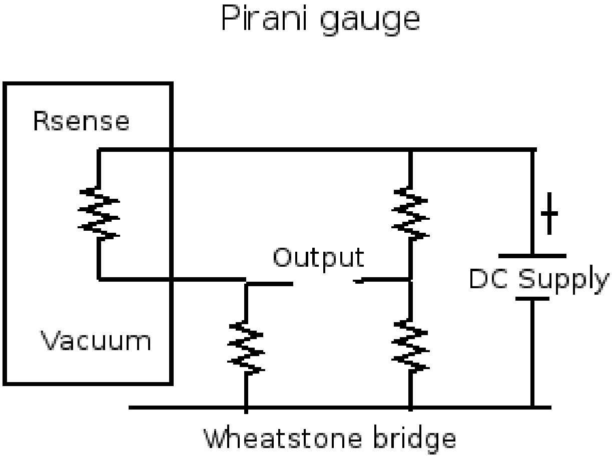
Low pressure sensors are called pressure *gauge*s.

Three Important Low Pressure Gauges:

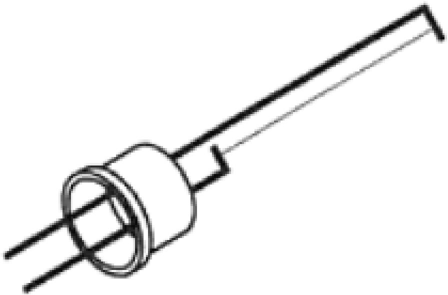
Type	Working Principle	Range
Pirani Gauge	Heat Convection	0.5 Torr to 10^{-4} Torr
Penning Gauge	Ionization: Cold cathode	10^{-2} mbar to 10^{-7} mbar
Bayard-Alpert Gauge	Ionization: Hot cathode	10^{-3} mbar to 10^{-10} mbar

Pirani Gauge

Thermal Convection: Lower pressure ⇒ Less heat carried away
⇒ More heating of resistor ⇒ Higher resistance

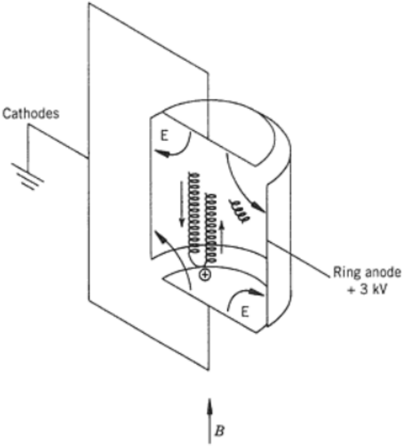


Pirani Gauge



Penning Gauge

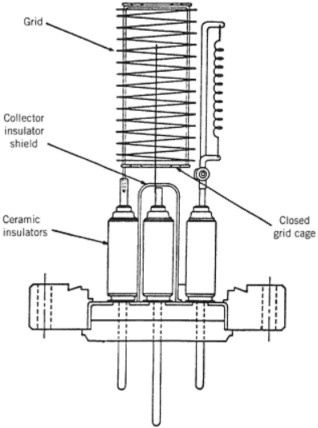
Cold cathode: Requires 2 to 4 kV
Starting problem at very low pressures.
Magnetic field often used for longer travel paths of electrons.



Trajectories and fields in the

Bayard-Alpert Gauge

Avoids x-ray ionization problem of the triode gauge
Hot cathode: Can measure very low pressures



Bayard-Alpert Gauge

Example potentials at the electrodes:

- Filament at +45 V
- Grid at +180 V
- Collector at 0 V

J. M. Lafferty: Foundations of vacuum science and technology
Wiley(1998)