IN 221 (AUG) 3:0
Sensors and Transducers
Lecture 3

A. Mohanty

Department of Instrumentation and Applied Physics (IAP)
Indian Institute of Science
Bangalore 560012

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Correction

The HX711 amplifier/digitizer board does not have the I^2C interface.
It uses a simpler interface.

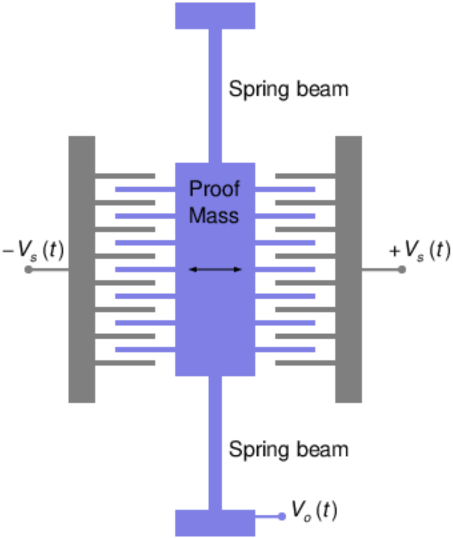
Importance of the Wheatstone bridge

- The working principle of many sensors involves the change in resistance of the sensing element.
- The Wheatstone bridge configuration is then used.
- Often, the output is very small.
- Amplification is needed.
- A *differential amplifier* must be used.
- An analogue-to-digital converter (ADC, or A/D) is often included so that the output is available in digital form.

Change of Capacitance Sensors

Many types of sensors use the change of resistance of the sensing elements.
Many other types of sensors use the *change of capacitance* of the sensing elements.
Examples are *accelerometers* and *pressure sensors*.

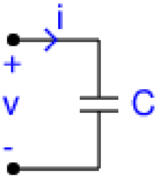
Accelerometer: Interdigitated Structure



Example Accelerometers with Interdigitated Structure

- ADXL335
- ADXL345
- MPU-6000 and MPU-6050

Capacitance



Q is the stored charge in the capacitor. $Q = Cv$.

$$C = \frac{Q}{v}$$

$$i = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathrm{d}(Cv)}{\mathrm{d}t} = C\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}C}{\mathrm{d}t}$$

If C is constant, $i = C\frac{\mathrm{d}v}{\mathrm{d}t}$.

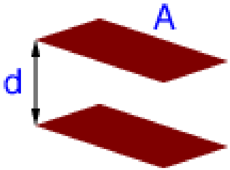
Sinusoidal Excitation

Let C be constant, and $v(t) = V_p \cos(2\pi ft) = V_p \cos(\omega t)$.
Here V_p is the *peak value* of the applied sinusoidal voltage. f is the *frequency* of the source, and ω is its *angular frequency*.
Then

$$i(t) = -2\pi fCV_p \sin(2\pi ft) = -\omega CV_p \sin(\omega t) \tag{1}$$

Peak value of the current is ωCV_p. It leads the voltage by a right angle.

Parallel Plate Capacitor

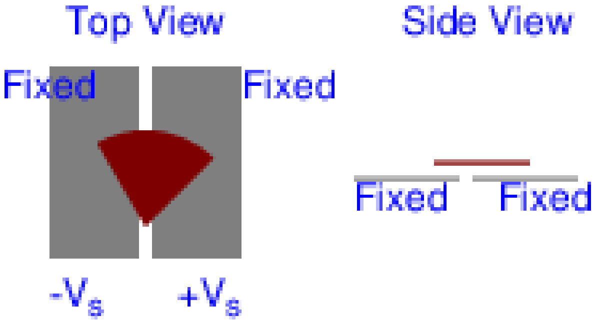


Capacitance

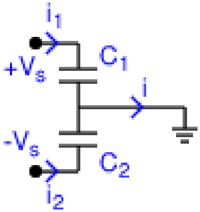
$$C = \frac{\epsilon A}{d} = \frac{\kappa \epsilon_0 A}{d} \tag{2}$$

Electric constant: $\epsilon_0 = 8.854\,187\,82 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
κ: Dielectric constant of the medium separating the plates.
κ is 1 for free space and air.
This formula neglects fringing field effects.

Capacitive Angle Sensor Electrodes



Output Current is Proportional to ΔC

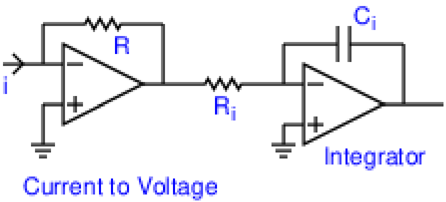


$$i = i_1 + i_2 = C_1 \frac{\mathrm{d}V_s}{\mathrm{d}t} + C_2 \frac{\mathrm{d}(-V_s)}{\mathrm{d}t} = (C_1 - C_2)\frac{\mathrm{d}V_s}{\mathrm{d}t} = \Delta C \frac{\mathrm{d}V_s}{\mathrm{d}t} \quad (3)$$

Note: The movable plate output is usually connected to a current to voltage converter.

The output is proportional to ΔC, which in turn is proportional to (i) the angle in case of the angle sensor, or (ii) the displacement of the proof mass in case of the MEMS accelerometer.

Circuit for Amplifier

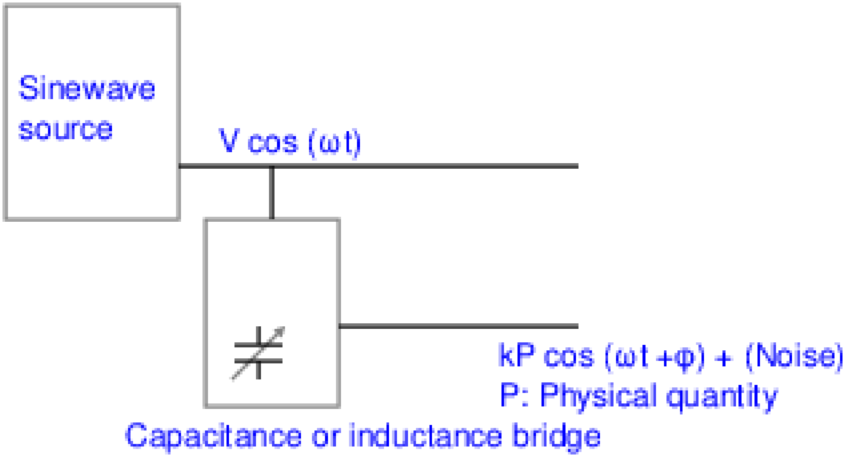


The integrator is optional.

Accelerometer or Pressure sensor

- A flexible electrode moves a small distance that is proportional to the acceleration or the pressure difference to be measured.
- This small movement changes a capacitance or an inductance.
- With a bridge arrangement, this sensor can be …
  - … sensitive,
  - … reliable,
  - … and quite linear.

How do we measure *P*?



How do we measure P?

Physical quantity to be measured: P
Output: (Signal) + (Noise)
Signal: $kP\cos(\omega t + \phi)$

$$v_o = kP\cos(\omega t + \phi) + (\text{Noise})$$

- The signal output may be quite small.
- It is still proportional to P.
- Here, ϕ is a constant phase shift that is specific to the bridge instrumentation.

Trouble with Rectification

- P can be negative.
- We need to extract it from the signal $kP\cos(\omega t + \phi)$.
- Rectification (full-wave or half-wave) will NOT work.

Trouble with simple amplification

- In industrial settings, the noise strength may be comparable to the signal. It may sometimes be greater.
- If we simply amplify the output, the noise will also be amplified.
- Filtering will decrease the noise, but that may not be enough.
- Cases like this are very common.

Lock-in Amplifier

- Solution: Use a *lock-in amplifier*.
- Other names:
  - Synchronous detection
  - Phase-sensitive detection
- Implementation: Multiply the bridge output v_o with the reference sinewave and average (low-pass filter) it.
- What is the resulting output?

Lock-in Amplifier Mathematics

The output of the multiplier is

$$V\cos(\omega t) \times kP\cos(\omega t + \phi) = kVP[\cos(\phi)\cos^2(\omega t) - \sin(\phi)\sin(\omega t)\cos(\omega t)]. \quad (4)$$

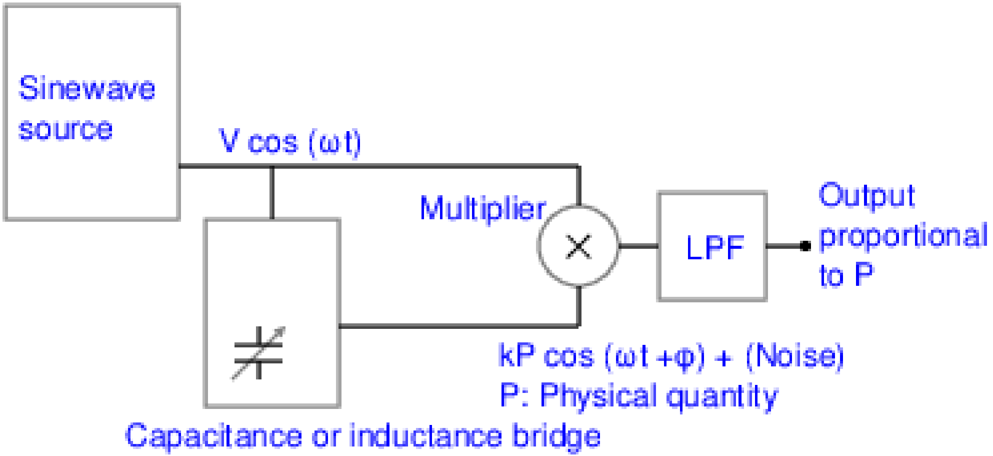
$$\cos^2(\omega t) = \frac{1}{2} + \frac{\cos(2\omega t)}{2}, \quad (5)$$

and

$$\sin(\omega t)\cos(\omega t) = \frac{\sin(2\omega t)}{2}. \quad (6)$$

Both $\cos(2\omega t)$ and $\sin(2\omega t)$ have average value of 0.
So the average value of $\cos^2(\omega t)$ is $\frac{1}{2}$, and that of $\sin(\omega t)\cos(\omega t)$ is 0.
The average value of the product is $\frac{1}{2}kVP\cos(\phi)$. $\Rightarrow$ (LPF output) $\propto P$.
(If we are unlucky to have $\cos\phi = 0$, we can use $V\sin(\omega t)$ instead of $V\cos(\omega t)$ at one input of the multiplier.)

Lock-in Amplifier: Block diagram

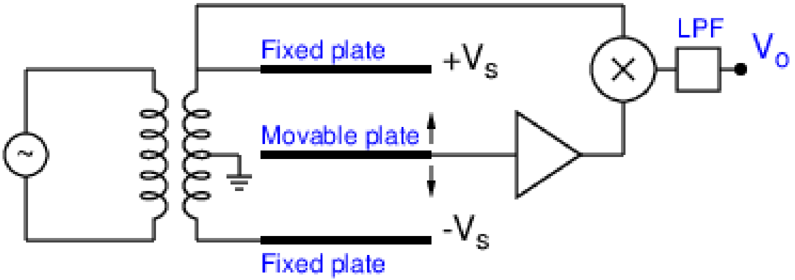


Demonstration of a Lock-in Amplifier

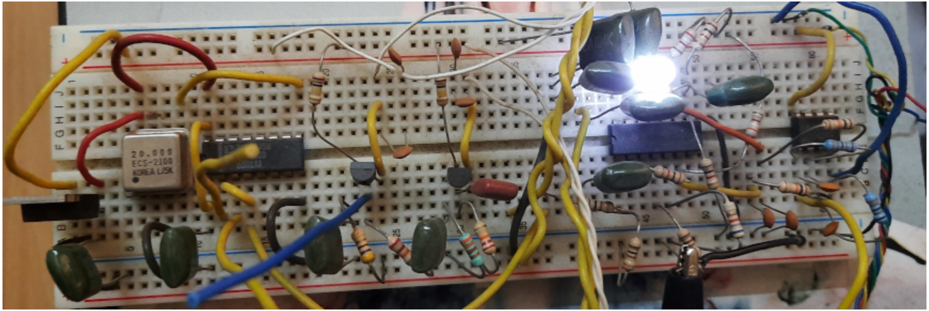
The following circuit shows how we may measure the difference between two small capacitances using a lock-in amplifier.

- Constructed on bread board.
- $\Delta C < 1\,\mathrm{pF}$
- Layout not neat at all …
- … yet the output signal is very clean.

Capacitive Sensor: Block Diagram



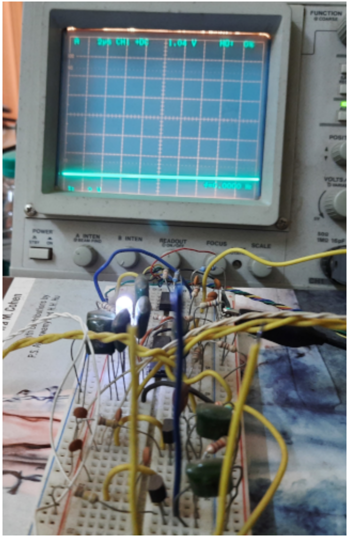
Example Differential Capacitive Sensor



Capacitive Sensor: Circuit Details

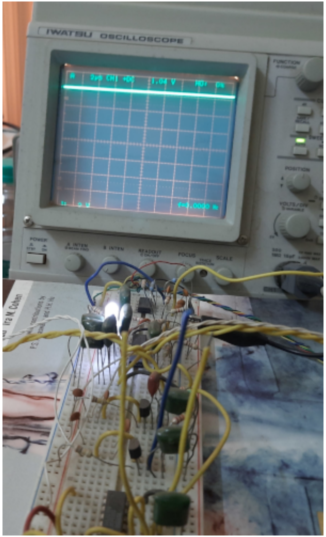
- 20 MHz crystal oscillator output given to a divide by 2 counter.
- 1/4 of 74LS175 D flip-flop used to implement the divide by 2 counter.
- Q_0 output is $+V_s(t)$, $\overline{Q}_0$ output is $-V_s(t)$
- The two yellow wires sticking out are the fixed plates.
- The blue wire sticking out is the movable plate.
- The amplifier is a two-transistor tuned amplifier.
- MC1496 is used as the multiplier.
- Uses RC LPF.
- NE5532 is the final amplifier after the LPF.

Blue Wire Closer to the Left Yellow Wire



Blue Wire Equidistant from the Yellow Wires

Blue Wire Closer to the Right Yellow Wire



MEMS Accelerometer

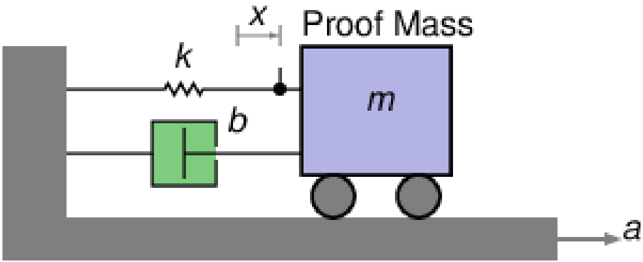
What is an accelerometer?

- A sensor whose output is proportional to its acceleration, at least when the acceleration is changing slowly
- If the acceleration is changing too fast, then the output may not be able to keep up with the input.
- Accelerometers usually have three output channels for the three components of acceleration.
- Some accelerometers provide analogue voltage outputs, while others internally convert their outputs to a digital form.
- Accelerometers made using MEMS technology are very successful.

Accelerometer: Applications

- Sensing the acceleration of aircraft and vehicles
- Sensing the orientation of hand-held devices using the acceleration due to gravity
- Sensing vibration in industrial applications

Accelerometer: Modelling



Simplified Mass-Spring-Dashpot model
x: Displacement of the mass from its equilibrium position in the frame of the accelerometer

$$m\ddot{x} = -kx - b\dot{x} - ma$$

$$m\ddot{x} + b\dot{x} + kx = -ma \tag{7}$$

Accelerometer: Transfer Function

$$T(s) = \frac{X(s)}{A(s)} = \frac{-1}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

Note: The negative sign is due to my sign convention.
Many books will indicate x in the reverse direction to not have this negative sign.

$$T(s) = \frac{-1}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}, \tag{8}$$

where,

$$\omega_0 = \sqrt{\frac{k}{m}}, \tag{9}$$

and

$$Q = \frac{\sqrt{km}}{b}. \tag{10}$$

Accelerometer: Low Frequency Response

For $\omega \ll \omega_0$,

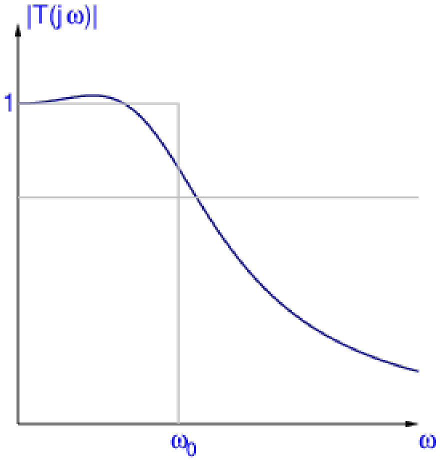
$$T(j\omega) \approx \frac{-1}{\omega_0^2} = -\frac{m}{k}. \tag{11}$$

Typical value: $f_0 = \dfrac{\omega_0}{2\pi} = 50\,\text{kHz}$

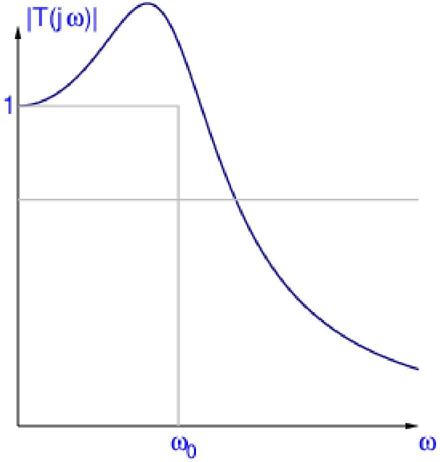
Conclusions

- For frequencies which are small compared to the resonant frequency, the displacement x is proportional to the acceleration.
- There is a need to make the electrical output proportional to x.
- In the MEMS accelerometer, for small x, $\Delta C \propto x$.

Accelerometer: Frequency Response for $Q = 0.8$



Accelerometer: Frequency Response for $Q = 1.2$



Accelerometer: Sensing the Displacement

Differential Capacitance Arrangement

- The proof mass acts as an electrode that moves between two other electrodes.
- It forms two capacitances that are equal when there is no displacement.
- The difference between the capacitances is proportional to the displacement, provided the displacement is small.

Challenge

- The devices are very small.
- The change in capacitance is very small.
- How do we reliably detect this small change?
- Answer: Use lock-in amplifier.