IN 221 (AUG) 3:0
Sensors and Transducers
Lecture 4

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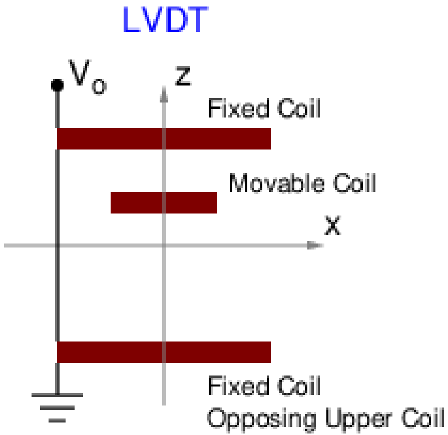
LVDT: Another Example of Synchronous Detection

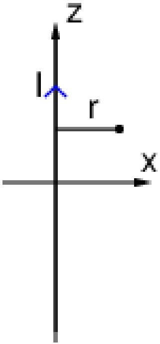
We have seen how synchronous detection helps in measuring small signals in sensors in which there is a small change of capacitance.
Now we discuss the LVDT, a displacement sensor that uses inductive sensing elements.

- Linear Variable Differential Transformer
- Two fixed coils in anti-series (in opposition)
- One movable coil
- Usually the movable coil is excited with sinewave AC.
- Fixed coil output is given to the phase sensitive detector.
- Quite robust and accurate

LVDT Coils


LVDT
Vo
Z
Fixed Coil
Movable Coil
X
Fixed Coil
Opposing Upper Coil

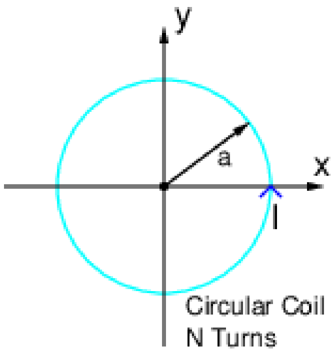
Magnetic Field due to Infinite Wire



$$B_\phi = \frac{\mu_0 I}{2\pi r} \tag{1}$$

Magnetic constant: $\mu_0 = 4\pi \times 10^{-7}$ T m / A
Example computation: If $I = 10$ A, and $r = 1$ cm, then $B_\phi = 0.2$ mT.

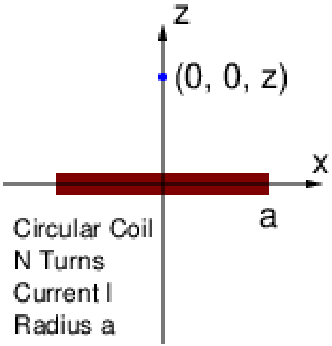
Circular Coil: B at Centre



$$B_z(0,0,0) = N\frac{\mu_0 I}{2a} \tag{2}$$

Example computation: If $N = 100$, $I = 1$ A, and $a = 5$ cm, then $B_z(0,0,0) = 1.2566$ mT.

Circular Coil: B on the Axis



$$B_z(0,0,z) = N\frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}} \tag{3}$$

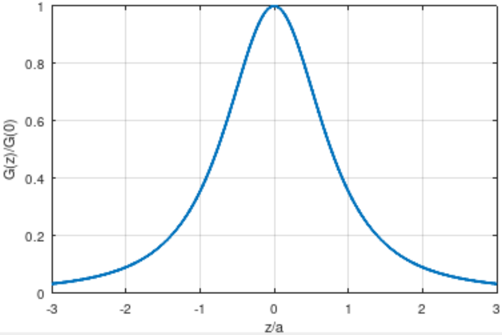
Example computation: If $N = 100$, $I = 1$ A, $a = 5$ cm, and $z = 5$ cm, then $B_z(0,0,z) = 0.444\,29$ mT.

Variation of B_z on the Axis

Let $G(z)$ be defined by

$$G(z) = B_z(0,0,z) = N\frac{\mu_0 I a^2}{2(a^2+z^2)^{3/2}}. \tag{4}$$

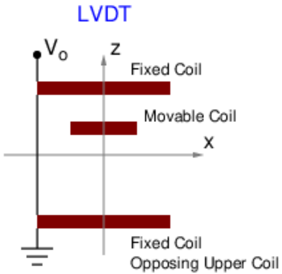
For a given coil, how does $G(z)$ vary with z? Here is a normalized plot.



Properties

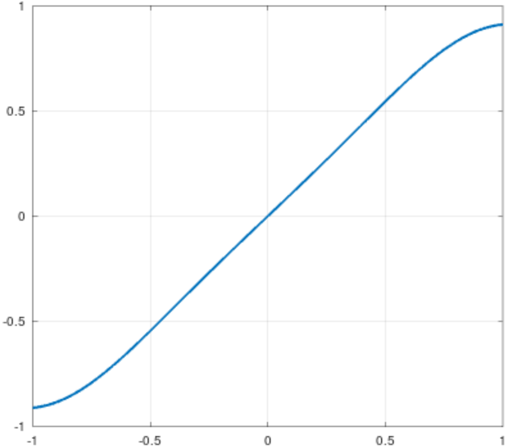
- Maximum at $z = 0$
- Even function: $G(z) = G(-z)$
- Has points of inflection ...
- ... near which variation is linear.

LVDT Field Analysis

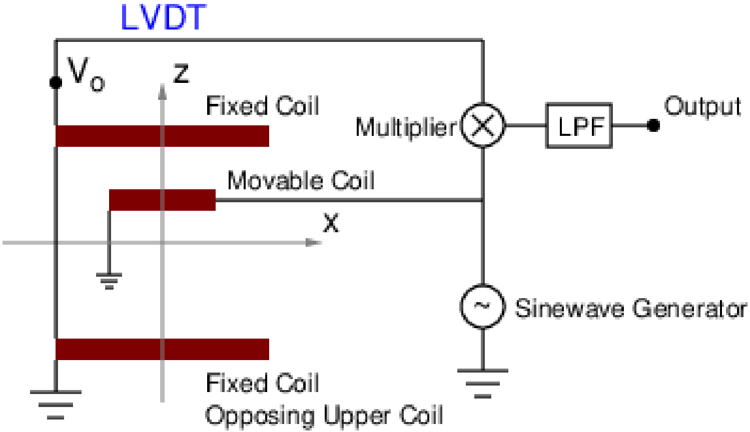


Let the distance from the centre to each coil be h.
Using calculus we can show that
$G(z-h) - G(z+h) = G(h-z) - G(h+z)$
is approximately $-2G'(h)z = Kz$ for small z.
If h is chosen as the height of the inflection point, the variation is very linear.

LVDT Output Linearity



LVDT Block Diagram

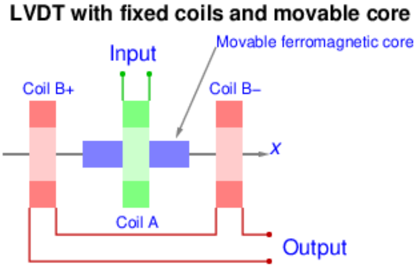


LVDT Analysis

Detailed analysis of the LVDT requires the study of the following concepts.

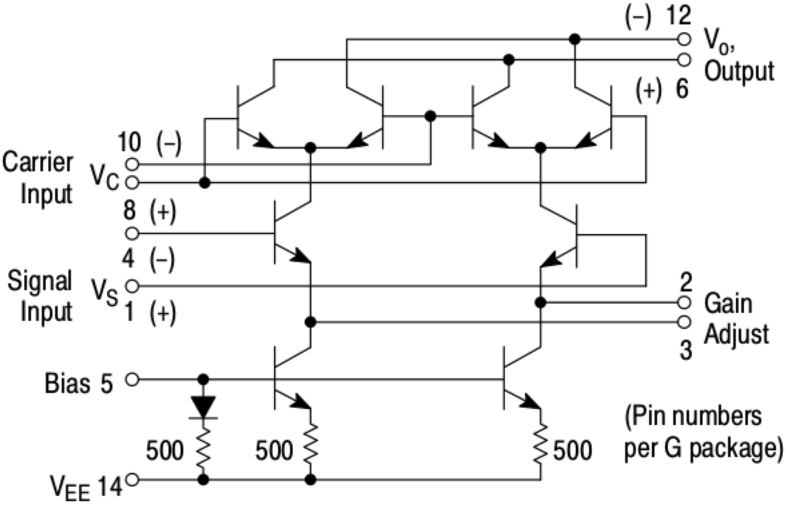
- Magnetic field of a coil
- Mutual inductance
- Lock-in amplifier

LVDT with fixed coils and movable core



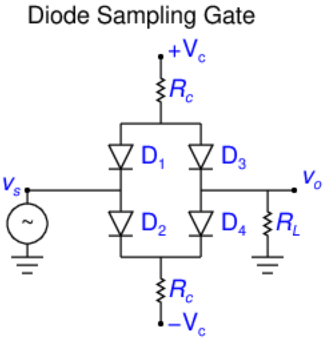
- Coils A, B$+$, and B$-$ are fixed.
- Fields due to B$+$ and B$-$ are in opposition.
- Many modern LVDTs are of this type.

Circuit for multiplication: Gilbert Cell



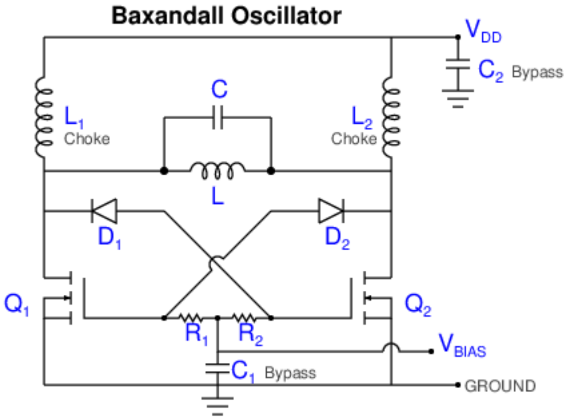
Gilbert cell used in MC1496

Circuit for multiplication: Sampling Gate



Diode sampling gate circuit: $v_o = v_s$, if V_c is able to turn the diodes on, otherwise $v_o = 0$.
If v_s and V_c are both sinusoids of the same frequency and V_c is large in amplitude, then v_o is proportional to the component of v_s along V_c.

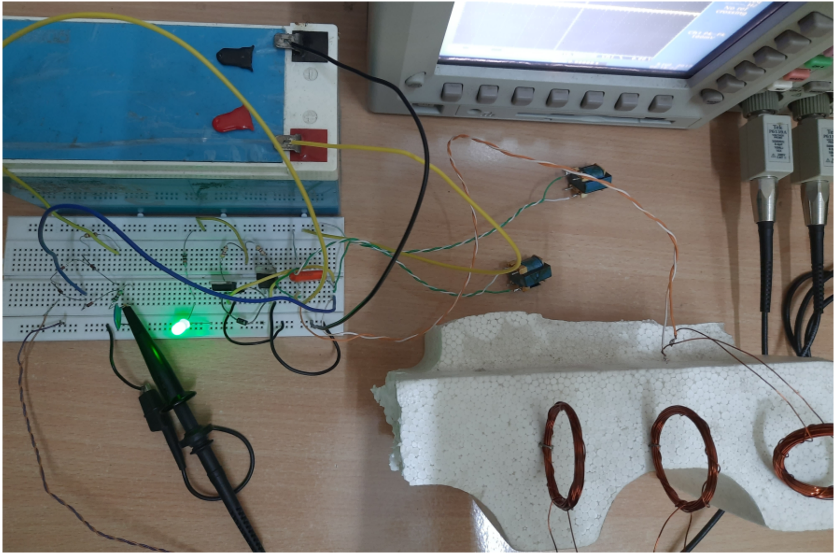
Baxandall Oscillator



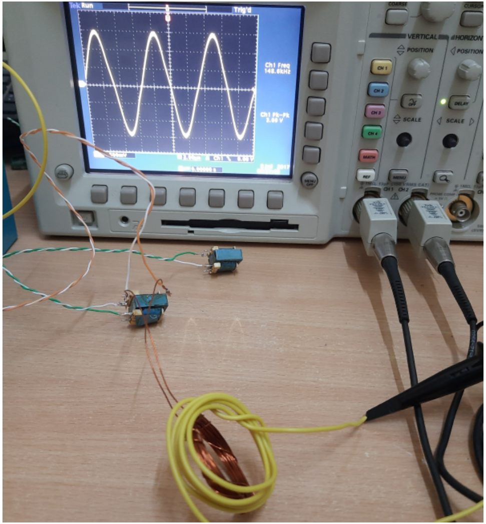
Baxandall Oscillator Parts

- Field coil $L = 97\,\mu\text{H}$
- $C = 10.29\,\text{nF}$
- MOSFETs: ST55NF06L or IRF540N
- Diodes: 1N4007
- $R_1 = R_2 = 470\,\Omega$
- Chokes: $L_1 = L_2 = 1\,\text{mH}$
- Bypass capacitors C_1, C_2 not used.
- $V_{\text{DD}} = V_{\text{BIAS}} = 12.0\,\text{V}$

LVDT Demo: System

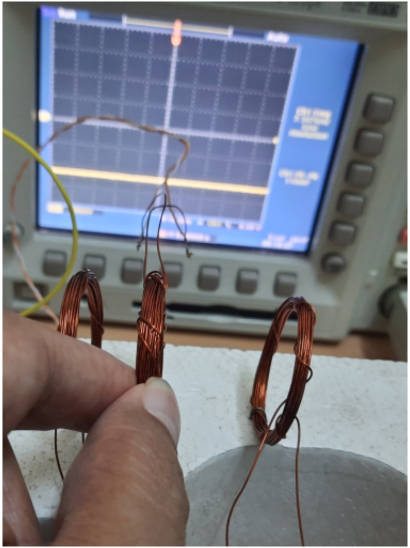


LVDT Demo: Oscillation

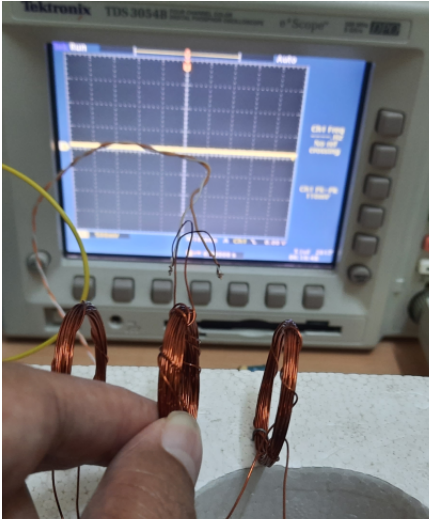


Frequency of oscillation: $f = 148.6\,\text{kHz}$.

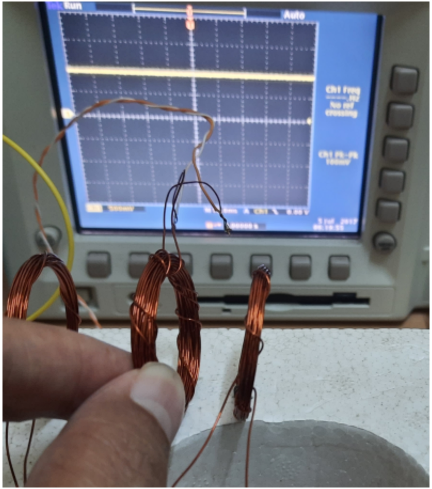
LVDT Demo: Coil moved left



LVDT Demo: Coil close to centre

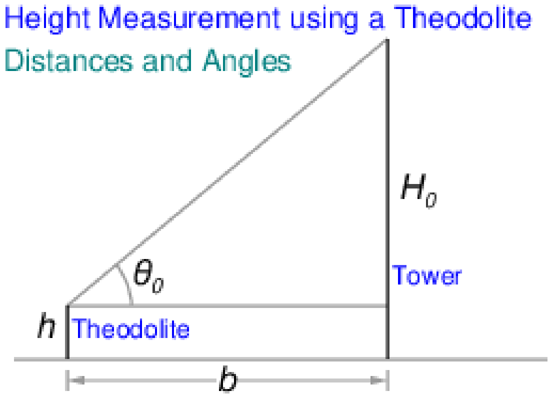


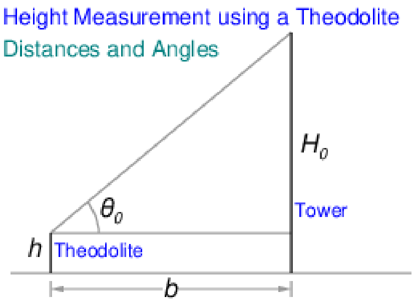
LVDT Demo: Coil moved right



Another Example of Error Analysis

We wish to measure the height of a tower using a theodolite.



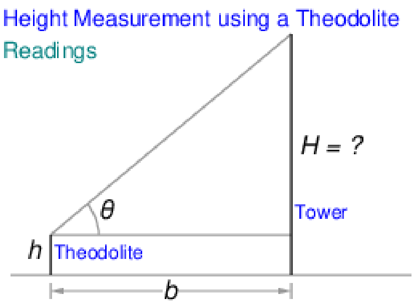


$$H_0 = h + b\tan\theta_0. \quad (5)$$

Note that

$$b = \frac{H_0 - h}{\tan\theta_0}. \quad (6)$$

Measurements



In actual measurement, the reading of the theodolite is θ, which is not the same as θ_0 due to instrument errors. Also, h and b can be measured very accurately. So we consider the angle (θ) measurement as the only source of error.
Estimated height is

$$H = h + b \tan \theta. \tag{7}$$

Error Analysis

Error in height is

$$H_0 - H = b(\tan\theta_0 - \tan\theta) \approx b\sec^2\theta_0 \times (\theta_0 - \theta).$$

But

$$b = \frac{H_0 - h}{\tan\theta_0}.$$

So

$$H_0 - H \approx b\sec^2\theta_0 \times (\theta_0 - \theta) = \frac{H_0 - h}{\tan\theta_0}\sec^2\theta_0 \times (\theta_0 - \theta).$$

Or,

$$H_0 - H \approx \frac{H_0 - h}{\sin\theta_0\cos\theta_0} \times (\theta_0 - \theta).$$

The error in angle measurement is magnified by a factor

$$K = \frac{H_0 - h}{\sin\theta_0\cos\theta_0} = 2\frac{H_0 - h}{\sin(2\theta_0)}. \quad (8)$$

What value of θ_0 minimizes K?

Answer

$\sin(2\theta_0)$ can be a maximum of 1, when $\theta_0 = \pi/4$.
So the answer is 45 degrees.
In other words, b should be as close to $H_0 - h$ as possible.

Types of questions that can be asked

- Numerical calculations
- Simple derivations
- System block diagrams