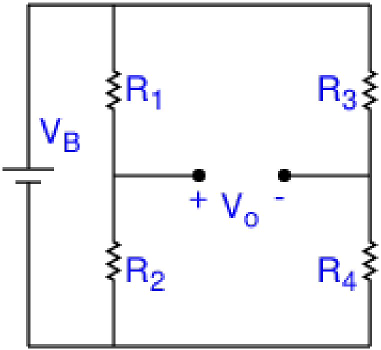
IN 221 (AUG) 3:0
Sensors and Transducers
Lecture 7

A. Mohanty

Department of Instrumentation and Applied Physics (IAP)
Indian Institute of Science
Bangalore 560012

29/08/2025

Practice Problem: Wheatstone Bridge



$$V_o = V_B\left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right) \quad (1)$$

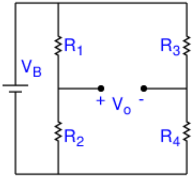
Practice Problems

1. The electrodes of a parallel plate capacitor are circular discs, each having a radius of 10 cm. If the electrodes are separated by an air gap of 1 mm, calculate the capacitance neglecting fringing fields.
2. A sine wave signal having peak voltage 20 V and frequency 10 MHz is applied across a 1 nF capacitor. Calculate the peak current in the capacitor.
3. A circular coil has 10 turns of wire with radius 10 cm. Calculate the magnitude of B on the axis of the coil at a distance 5 cm from the centre of the loop due to a 10 A current in the coil. Assume that there are no magnetic materials near the coil.
4. A resistor constructed using platinum wire has resistance 100 Ω at 20 ℃. What will be its resistance at 10 ℃?
5. In the Wheatstone bridge shown in the previous slide, $V_B = 10\,\text{V}$, $R_1 = R_4 = 100\,\Omega$, and $R_2 = R_3 = 96\,\Omega$. Calculate V_0.

Answers to Practice Problems

1. $278.157\,\text{pF}$
2. $1.256\,64\,\text{A}$
3. $44.9588\,\mu\text{T}$
4. $96.075\,\Omega$
5. $-204.082\,\text{mV}$

Practice Problem: Balanced Wheatstone Bridge



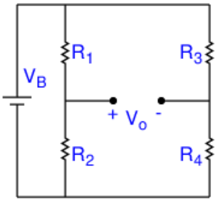
$$V_o = V_B\left(\frac{R_2}{R_1+R_2} - \frac{R_4}{R_3+R_4}\right) \tag{2}$$

When is $V_o = 0$?

$$V_o = V_B\frac{R_2R_3 + R_2R_4 - R_1R_4 - R_2R_4}{(R_1+R_2)(R_3+R_4)} = V_B\frac{R_2R_3 - R_1R_4}{(R_1+R_2)(R_3+R_4)}. \tag{3}$$

For $V_o = 0$, it is required to have $R_2R_3 = R_1R_4$.

Balanced Wheatstone Bridge



Condition for $V_o = 0$:

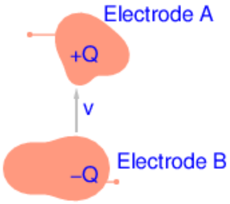
$$\frac{R_1}{R_2} = \frac{R_3}{R_4}. \tag{4}$$

Alternately,

$$\frac{R_1}{R_3} = \frac{R_2}{R_4}. \tag{5}$$

When $V_o = 0$, with $V_B \neq 0$, the Wheatstone bridge is said to be *balanced*.

Capacitor



Capacitor Electrodes

- Two conducting electrodes
- Capable of storing charge

$$Q = Cv. \quad (6)$$

Current in a capacitor

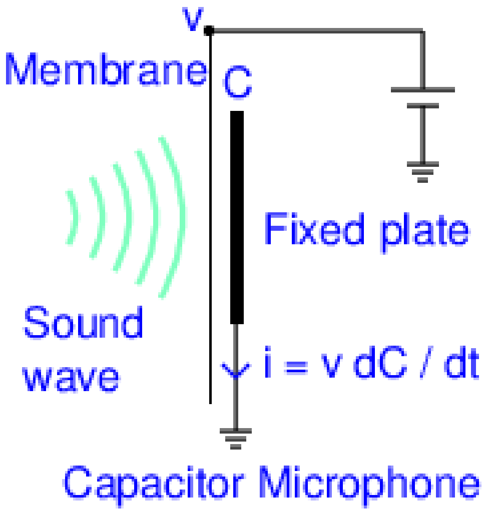
$$i = \frac{\mathrm{d}Q}{\mathrm{d}t} = \frac{\mathrm{d}Cv}{\mathrm{d}t} = C\frac{\mathrm{d}v}{\mathrm{d}t} + v\frac{\mathrm{d}C}{\mathrm{d}t}. \tag{7}$$

Current for fixed C: Usually C is fixed or changes very slowly. Then

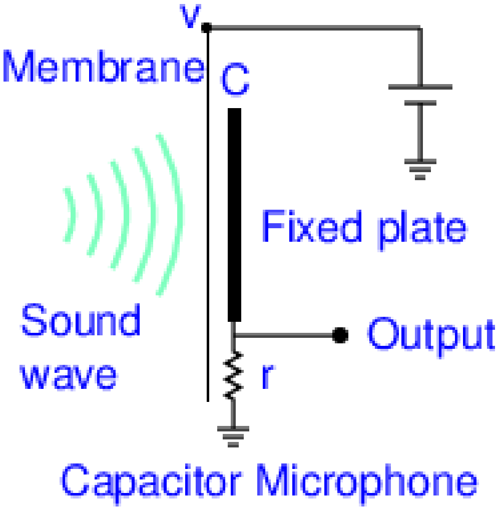
$$i = C\frac{\mathrm{d}v}{\mathrm{d}t}. \tag{8}$$

But there is a very important class of sensors in which the $v\frac{\mathrm{d}C}{\mathrm{d}t}$ term is used.

Capacitor Microphone



Voltage Output from Capacitor Microphone



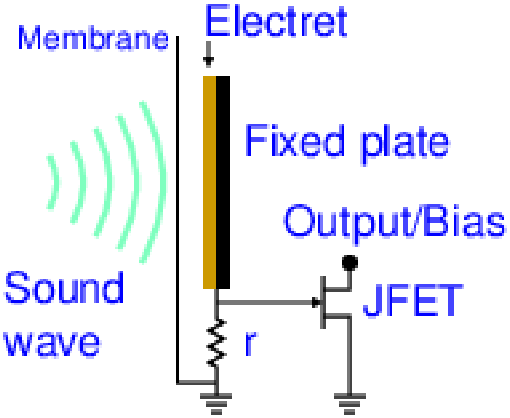
Capacitor Microphone Characteristics

- Advantages: High fidelity, directionality
- Disadvantage: Requires extra power source High voltage may be required.

Electret

- Permanently polarized dielectric material
- Electrical equivalent of a magnet
- Can provide a large electric field to replace the battery in a capacitor microphone

Electret Microphone



Electret Microphone

Energy Stored in a Capacitor

Charge stored:

$$Q = Cv. \tag{9}$$

Current:

$$i = \frac{\mathrm{d}Q}{\mathrm{d}t} = C\frac{\mathrm{d}v}{\mathrm{d}t}. \tag{10}$$

Power:

$$p = vi = Cv\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}\left(\frac{1}{2}Cv^2\right)}{\mathrm{d}t}. \tag{11}$$

Energy stored:

$$U = \frac{1}{2}Cv^2 = \frac{1}{2}C\left(\frac{Q}{C}\right)^2 = \frac{1}{2}\frac{Q^2}{C}. \tag{12}$$

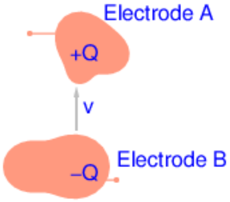
U was expressed in terms of charge Q, because in an isolated capacitor, Q is a constant.

Reconfiguration

What happens when we *reconfigure*, that is change the position and/or orientation, of the electrodes of a capacitor?

- The capacitance C, and the stored energy U change.
- If the capacitor is isolated, the stored charge Q does *not* change.
- This can be used to derive the *force* and the *torque* exterted by an electrode of a charged capacitor.
- In this study, we only derive an expression for the force.

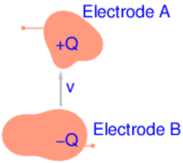
Force exterted by an electrode



Capacitor Electrodes

The figure shows electrodes of a capacitor which is charged to charge Q. Irregular shapes are shown, because this is part of a general derivation that is not specific to any standard type of capacitor.
Let one of the electrodes, say Electrode A, be considered movable.

Force exterted by an electrode



Capacitor Electrodes

Quantities like C, and U are now functions of the position and the orientation of Electrode A.
Assume that the orientation is fixed. Let the position of Electrode A be specified by coordinates x, y, and z of marked point on it. Then

$$C = C(x, y, z), \tag{13}$$

and

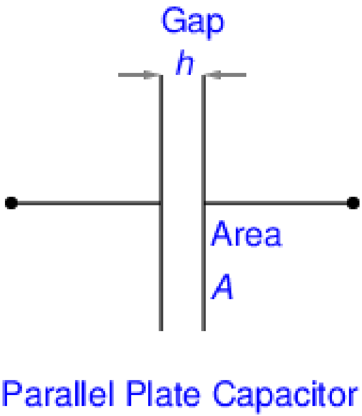
$$U = U(x, y, z). \tag{14}$$

Force exterted by an electrode

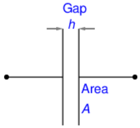
Let the force exterted by Electrode A, when it is held in place, be $\vec{F}$.
Changing the position of the electrode by a small displacement $\Delta\vec{r}$ would require work $-\vec{F}\cdot\Delta\vec{r}$ to be done on the system.
If the capacitor is isolated, this work would be added to the stored energy of the capacitor. So we have

$$\vec{F} = -\,\mathbf{grad}\,U = -\,\mathbf{grad}\left(\frac{1}{2}\frac{Q^2}{C}\right) = \frac{1}{2}\frac{Q^2}{C^2}\,\mathbf{grad}\,C = \frac{1}{2}v^2\,\mathbf{grad}\,C. \qquad (15)$$

Force in a parallel plate capacitor



Force in a parallel plate capacitor



Parallel Plate Capacitor

For the parallel plate capacitor shown,

$$C = \frac{\epsilon_0 A}{h}. \tag{16}$$

$$\textbf{grad}\, C = -\frac{\epsilon_0 A}{h^2}\hat{\mathbf{h}}, \tag{17}$$

rate of change of C in directions perpendicular to h being 0. On the electrode on the right,

$$\vec{F} = \frac{1}{2}v^2\,\textbf{grad}\, C. = -\frac{1}{2}\frac{\epsilon_0 A v^2}{h^2}\hat{\mathbf{h}}. \tag{18}$$

the negative sign indicating a force to the left.

Dependence on gap and voltage

$$\vec{F} = -\frac{1}{2}\frac{\epsilon_0 A v^2}{h^2}\hat{\mathbf{h}}.$$

- Negative sign indicates attaction between the plates
- Inversely proporional to the square of the gap: more effective for small devices (MEMS)
- Proportional to the square of the voltage: non-linear scale, true RMS

Electrostatic Actuators

- MEMS actuators
  - Used in projectors
  - Fast switches for RADAR and RF applications
- Electrostatic loudspeakers
  - High fidelity
  - Bias voltage required for linearity
- True RMS high-voltage voltmeters