IN 277 Notes 3
Generalized Impedance Converter (GIC)

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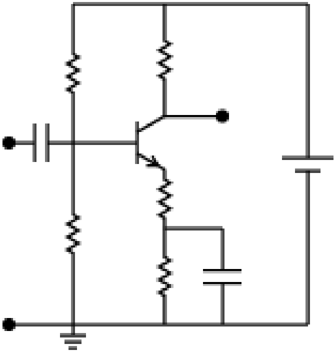
Artificial Inductors

- Classical filter design requires resistors, capacitors, and inductors.
- Inductors required for audio frequency filters are in the millihenry or higher ranges.
- Such large inductors are expensive, with high losses and nonlinearities.
- Using active circuits, it is possible to synthesize large low-loss inductors for filter work.
- Such *artificial inductors* have transformed telephony, and high fidelity audio design.
- However, in high power applications, they cannot replace physical inductors.

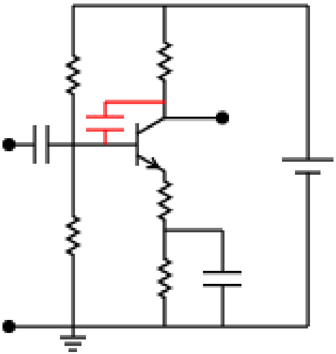
The Miller Effect

- Consider an impedance connected between the output and the input.
- May be a stray capacitance, or may be connected by design
- Usually harmful and degrades high frequency gain
- But here it is going to be used for realizing high quality grounded inductors.

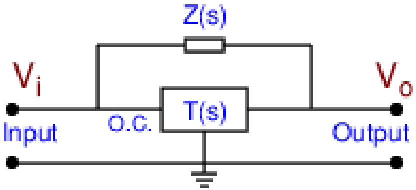
CE Amplifier



CE Amplifier: Stray Capacitance



Miller Effect: Block Diagram



Input Current: I_i

$$I_i = \frac{V_i - V_o}{Z(s)} = \frac{V_i - V_i T(s)}{Z(s)} = V_i \frac{1 - T(s)}{Z(s)}$$

Input Impedance:

$$Z_i = \frac{V_i}{I_i} = \frac{Z(s)}{1 - T(s)}$$

If the input of the $T(s)$ block is not an open circuit, then that input impedance would be in parallel with Z_i.
For now, we focus on Z_i.

Miller Effect: Inverting Amplifier

Input Impedance:

$$Z_i = \frac{Z(s)}{1 - T(s)}$$

Inverting Amplifier like BJT CE Amplifier: $T(s) = -K$

$$Z_i = \frac{Z(s)}{1 + K}$$

If K is large, $|Z_i| \ll |Z(s)|$, which can load the input a lot. If $K = 50$, $Z_i = Z(s)/51$. A 2 pF stray capacitance will look like a 102 pF load to the input.

Miller Effect: Non-inverting Amplifier

Input Impedance:

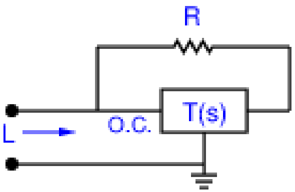
$$Z_i = \frac{Z(s)}{1 - T(s)}$$

Non-inverting Amplifier: $T(s) = +K$

$$Z_i = -\frac{Z(s)}{K - 1}$$

If K is large, we still have $|Z_i| \ll |Z(s)|$, which can load the input a lot. In addition, due to the negative sign, stray capacitances look like inductive loads to the input. This may lead to various circuit instabilities.

Miller Effect: Making a Grounded Inductor



$$Z_i = \frac{Z(s)}{1 - T(s)}$$

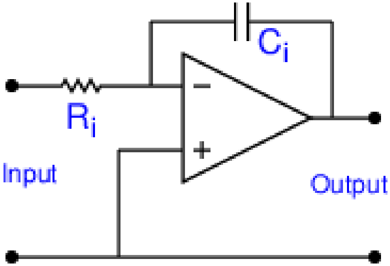
Let $Z(s) = R$, and the desired input impedance be purely inductive, that is $Z_i = sL$. Then we require

$$T(s) = 1 - \frac{Z(s)}{Z_i} = 1 - \frac{R}{sL} = 1 - \frac{1}{\tau s}$$

where $\tau = L/R$.

What is $1 - 1/(\tau s)$?

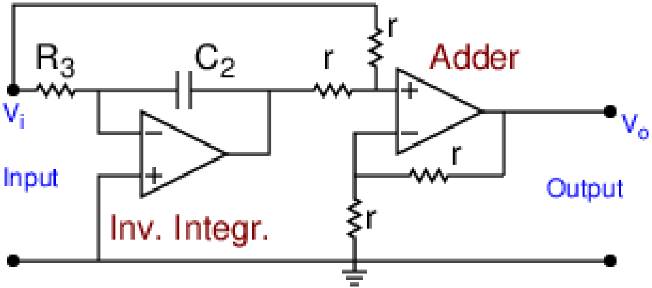
We recall that $-\dfrac{1}{\tau s}$ is the transfer function of an inverting integrator, where $\tau = R_i C_i$ is the time constant of the integrator.



So the desired transfer function is the transfer function of an inverting integrator plus 1. If τ is the time constant of the inverting integrator, then

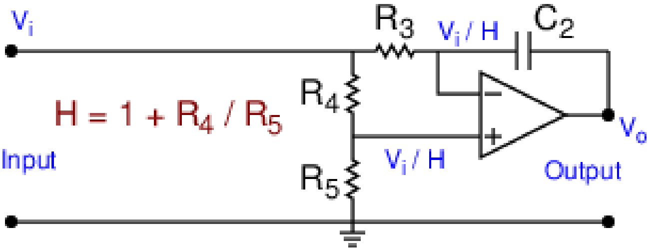
$$L = R\tau$$

Realizing $T(s)$ Using Adder



We still need to connect a unity follower buffer at the input.
Needs three operational amplifiers including the unity gain buffer.

Realizing $T(s)$: Modifying the Integrator



Here

$$\frac{V_i - V_i/H}{R_3} = (V_i/H - V_o)sC_2$$

Or,

$$\frac{V_i - V_i/H}{sR_3C_2} = V_i/H - V_o$$

Realizing $T(s)$: Modifying the Integrator

Or,

$$V_o = V_i/H - \frac{V_i - V_i/H}{sR_3C_2}$$

Or,

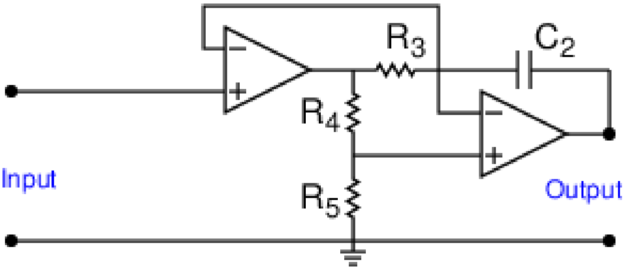
$$\frac{V_o}{V_i} = \frac{1}{H} - \frac{1 - 1/H}{sR_3C_2}$$

which is *not* in the form desired. However,

$$\frac{V_o}{V_i/H} = 1 - \frac{H-1}{sR_3C_2}$$

which is in the correct form, with $\tau = R_3C_2/(H-1)$.
To make V_i/H the effective input, we need to connect the inverting input of the buffer operational amplifier to a point where the potential is V_i/H, while connecting its output to the point where the potential is V_i.

Realizing $T(s)$: Buffering the Input

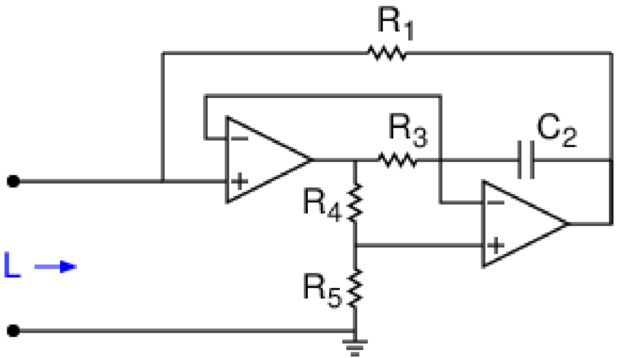


$$T(s) = 1 - \frac{1}{s\tau}$$

where $\tau = \dfrac{R_3 C_2}{H - 1}$, with $H = 1 + \dfrac{R_4}{R_5}$.

So $\tau = \dfrac{C_2 R_3 R_5}{R_4}$.

An Ideal Grounded Inductor



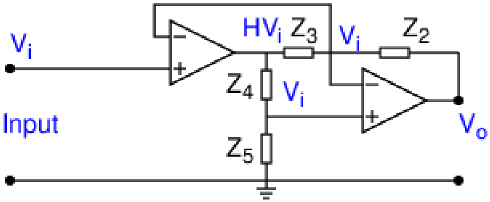
$$L = R_1\tau = \frac{R_1 C_2 R_3 R_5}{R_4}$$

This circuit is an example of a Generalized Impedance Converter (GIC).

GIC Inductor Uses

- Advantage: Can be used as a lossless inductor in filters
- Advantage: Can be very compact
- Advantage: Can give very large inductance values
- Disadvantage: Cannot carry high current
- Disadvantage: Cannot be used as an energy storage element

Circuit with General Impedances

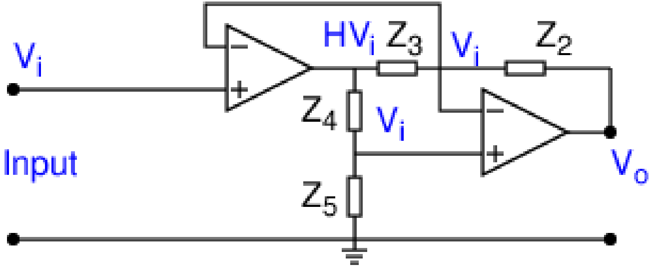


$$H = 1 + \frac{Z_4}{Z_5}$$

$$(HV_i - V_i)/Z_3 = (V_i - V_o)/Z_2$$

$$V_i(H-1)Z_2/Z_3 = V_i - V_o$$

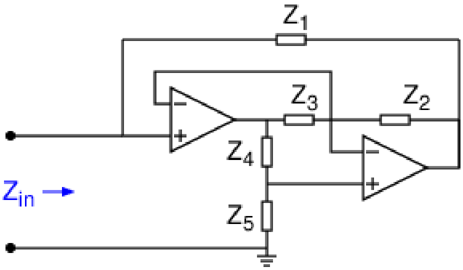
Circuit with General Impedances: Analysis



$$V_o = V_i - V_i(H-1)Z_2/Z_3$$

$$T(s) = \frac{V_o}{V_i} = 1 - (H-1)\frac{Z_2}{Z_3} = 1 - \frac{Z_4}{Z_5}\frac{Z_2}{Z_3} = 1 - \frac{Z_2 Z_4}{Z_3 Z_5}$$

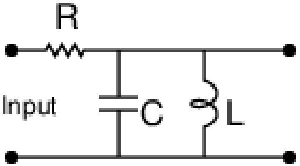
Generalized Impedance Converter (GIC)



$$Z_{\text{in}} = \frac{Z_1}{1 - T(s)} = \frac{Z_1}{\frac{Z_2 Z_4}{Z_3 Z_5}} = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4}$$

This circuit is useful for creating exotic impedances that are not realizable with normal RLC circuits.

A BPF Using a Grounded Inductor

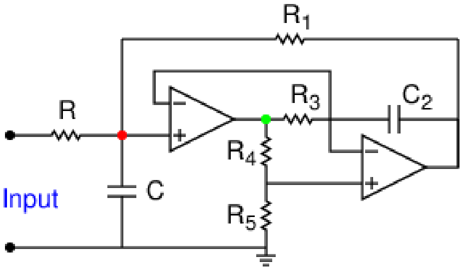


$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \frac{R}{\sqrt{L/C}}$$

$$H = 1$$

BPF Using the Ideal Grounded Inductor



Where should we connect the load? If we use the terminal marked by the red dot, that would correspond to the classical parallel RLC BPF. But that terminal is not buffered. So loading it will alter the transfer function.
The terminal marked by the green dot is buffered, but it provides a magnified version of the signal at the red dot. So we need to use the terminal with the green dot as the output and then do gain reduction by splitting the input resistor R.

GIC BPF Design

One usually selects $R_4 = R_5$, so that $H = 1 + R_4/R_5 = 2$, $R_1 = R_3 = r$, and $C_2 = C$.
Then $L = r^2 C$.
Then

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{r^2 C^2}} = \frac{1}{rC}$$

and

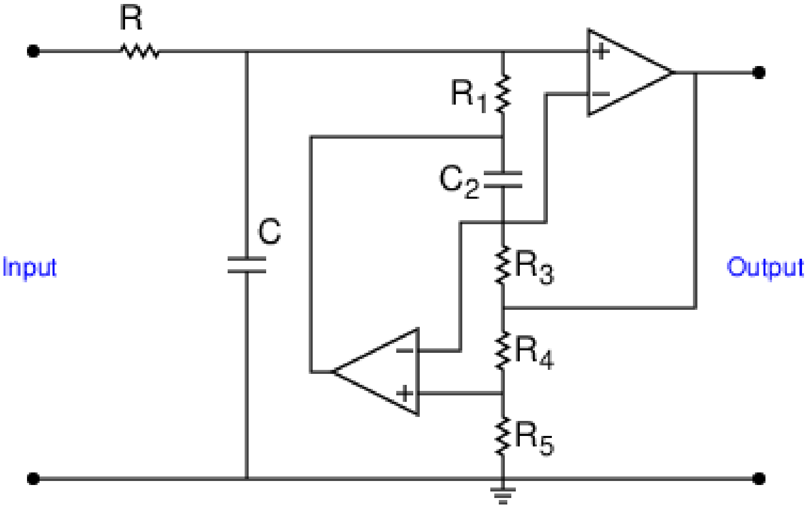
$$Q = \frac{R}{\sqrt{L/C}} = \frac{R}{\sqrt{r^2}} = \frac{R}{r}$$

So given f_0, Q and C, first we compute $r = 1/(2\pi f_0 C)$, and then $R = Qr$.

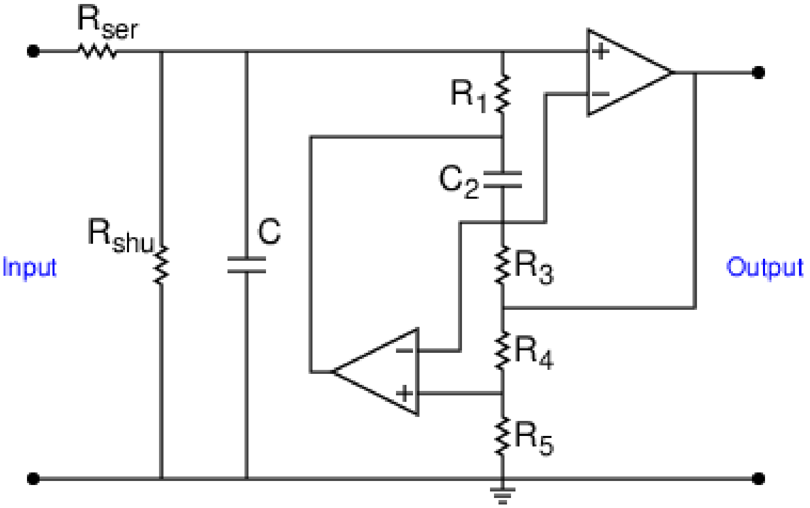
GIC BPF Design Gain Reduction

The output taken at the green dot would have a gain of 2 with this arrangement, so require $a = 1/2$.
This means that we need $R_{\text{ser}} = R/a = 2R$, and $R_{\text{shu}} = R/(1 - a) = 2R$.

GIC BPF Redrawn



GIC BPF With Gain Reduction



GIC BPF Design Summary

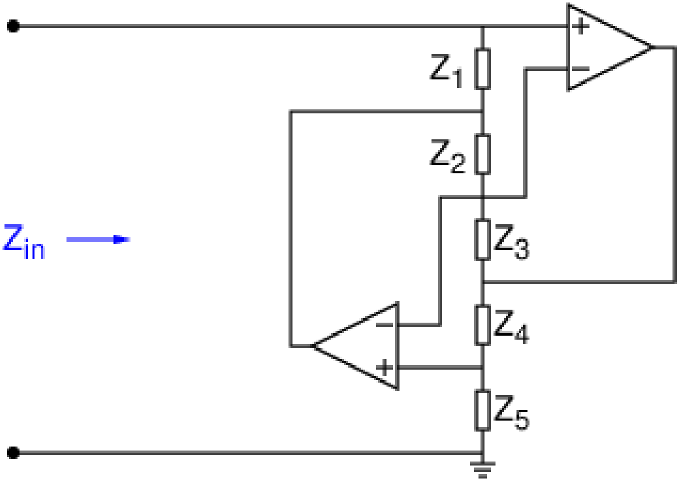
Specifications: f_0, Q, C, and desired gain of 1.

1. Compute $r = 1/(2\pi f_0 C)$.
2. Compute $R = Qr$.
3. Compute $R_{\mathrm{ser}} = 2R$.
4. Compute $R_{\mathrm{shu}} = 2R$.
5. Set $R_4 = R_5$. Both can be some standard value, say $10\,\mathrm{k}\Omega$.
6. Set $R_1 = R_3 = r$.
7. Set $C_2 = C$.

Remarks on the GIC BPF

- Works well in practice.
- Can be adjusted for high Q use.
- $Q = 50$ is possible.

GIC Redrawn



Frequency Dependent Negative Resistor (FDNR)

Let us select, $Z_1 = 1/(sC_1)$, $Z_2 = R_2$, $Z_3 = R_3$, $Z_4 = R_4$, and $Z_5 = 1/(sC_5)$. Then

$$Z_{\text{in}}(s) = \frac{Z_1 Z_3 Z_5}{Z_2 Z_4} = \frac{R_3}{s^2 C_1 C_5 R_2 R_4}$$

Let $C_1 = C_5 = C$, and $R_2 = R_3$. Then

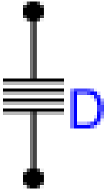
$$Z_{\text{in}}(s) = \frac{1}{s^2 C^2 R_4} = \frac{1}{s^2 D}$$

where $D = C^2 R_4$.

$$Z_{\text{in}}(j\omega) = -\frac{1}{\omega^2 D}$$

So the name is justified. This element helps in the design of ladder filters.

FDNR Symbol

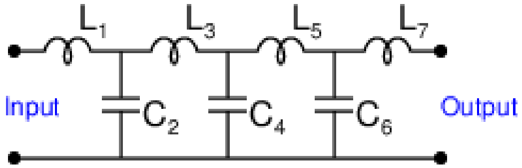


FDNR Impedance: $Z = \dfrac{1}{s^2 D}$

FDNR Impedance for $s = j\omega$: $Z = -\dfrac{1}{\omega^2 D}$

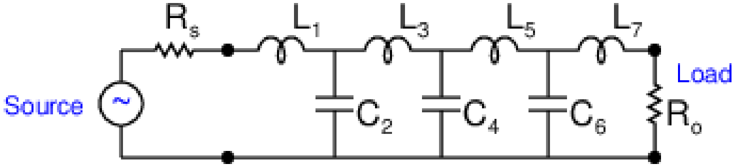
The symbol is made to look to like a double capacitor because of the double s, that is the s^2, in the denominator.

LC Ladder Lowpass Filter



Example LC Ladder Lowpass Filter

One way of designing passive lowpass filters results in an LC ladder network. An example is shown here.



LC Ladder Lowpass Filter in Use

Scaling of Impedances

In a network, if all impedances are scaled by the same factor, the transfer function remains unchanged.

Bruton Transformation

Named after Prof. Leonard T. Bruton of the University of Calgary.

Inductor: In order to avoid inductors, Bruton decided to scale all impedances by a factor $\frac{1}{\tau s}$, so that an inductor L with impedance sL is replaced by a resistance of value $R = \frac{L}{\tau}$.

Here τ is a conveniently chosen time constant.

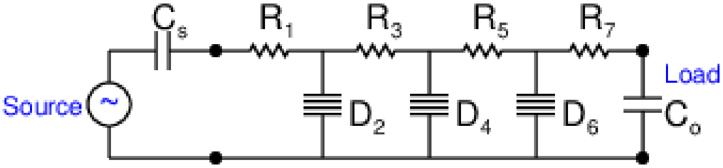
Resistor: Then a resistor R should be replaced by an element whose impedance is $\frac{R}{\tau s} = \frac{1}{s\tau/R}$. So a resistor R has be replaced by a capacitor which has value $C = \tau/R$.

Capacitor: A capacitor C whose impedance is $\frac{1}{sC}$ should be replaced by an element whose impedance is $\frac{1}{s^2\tau C}$. This is nothing but an FDNR with value $D = \tau C$.

Bruton Transformation Summary

Original Element	New Element
Inductor L	Resistor $R = L/\tau$
Resistor R	Capacitor $C = \tau/R$
Capacitor C	FDNR $D = \tau C$

Bruton Transformation Example Network



Bruton Transformation of the LC Ladder Lowpass Filter

Calculation of values: $C_s = \tau / R_s$, $R_1 = L_1/\tau$, $D_2 = \tau C_2$, $R_3 = L_3/\tau$, $D_4 = \tau C_4$, $R_5 = L_5/\tau$, $D_6 = \tau C_6$, $R_7 = L_7/\tau$, $C_o = \tau / R_o$.

τ should be chosen so that the resulting values are practical.

Each FDNR element can be implemented using the GIC circuit.