Non-isolated DC-DC Switched-mode Converters

November 21, 2025

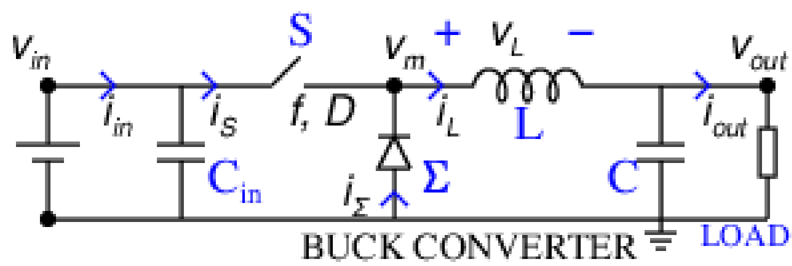

Figure 1: Buck converter.

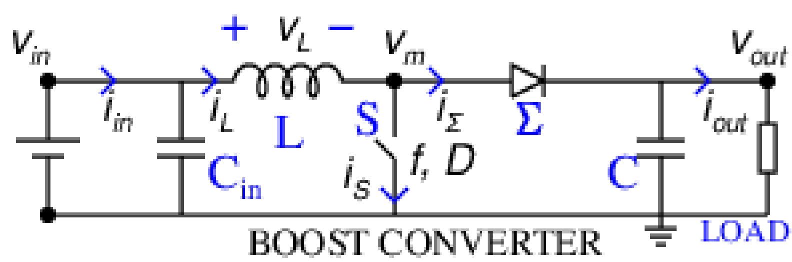

Figure 2: Boost converter.

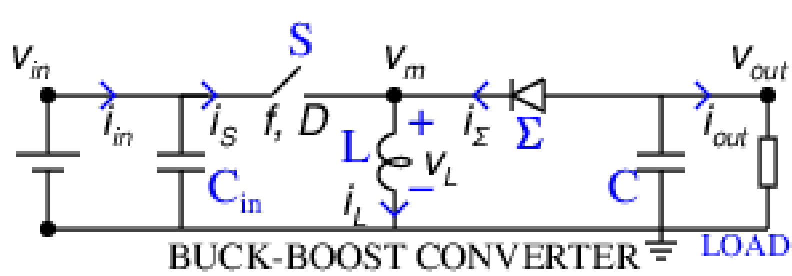

Figure 3: Inverting buck-boost converter.

1 Buck, Boost, and Buck-boost Converters

There are three main types of non-isolated DC-DC switched mode power converters. The buck converter, shown in Fig. 1, provides an output voltage that is smaller than the input voltage. The boost converter, shown in Fig. 2, provides an output voltage that is greater than the input voltage. The inverting buck-boost converter, shown in Fig. 3, provides a negative output voltage whose magnitude could be smaller or greater than the input voltage.

The most important elements for understanding these converters are the inductor L, the active switch S, and the diode (passive switch) Σ. The output capacitor C is assumed to be large enough to keep the output ripple voltage small, but it is not part of the most basic analysis. The input capacitor C_{in} is only used to reduce the ripple in the input current. It may be omitted in explaining the circuits.

In these converters, the active switch S is swichted on and off periodically with frequency f (period $T = 1/f$), and duty factor D. In each period, S is turned on for duration DT and then turned off for duration $(1 - D)T$. When S is on, Σ is off, and the inductor current increases in a ramp. When S is off, Σ is on while the inductor current decreases in a ramp. In the **CCM** (Continuous Conduction Mode), Σ is on whenever S is off, and the decreasing inductor current never quite reaches zero. In the **DCM** (Discontinuous Conduction Mode), Σ is on immediately after S is turned off, but the decreasing inductor current reaches zero in time δT that is less than $(1 - D)T$. Since Σ is a diode, it cannot allow the inductor current to be negative, it turns off leaving all currents to be zero at the end of the period. Fig. 4 contrasts the current waveforms for CCM and DCM. Whether the converter will be in CCM or DCM is determined by the current taken by the load. The converter enters DCM when the load takes less than a critical amount of current.

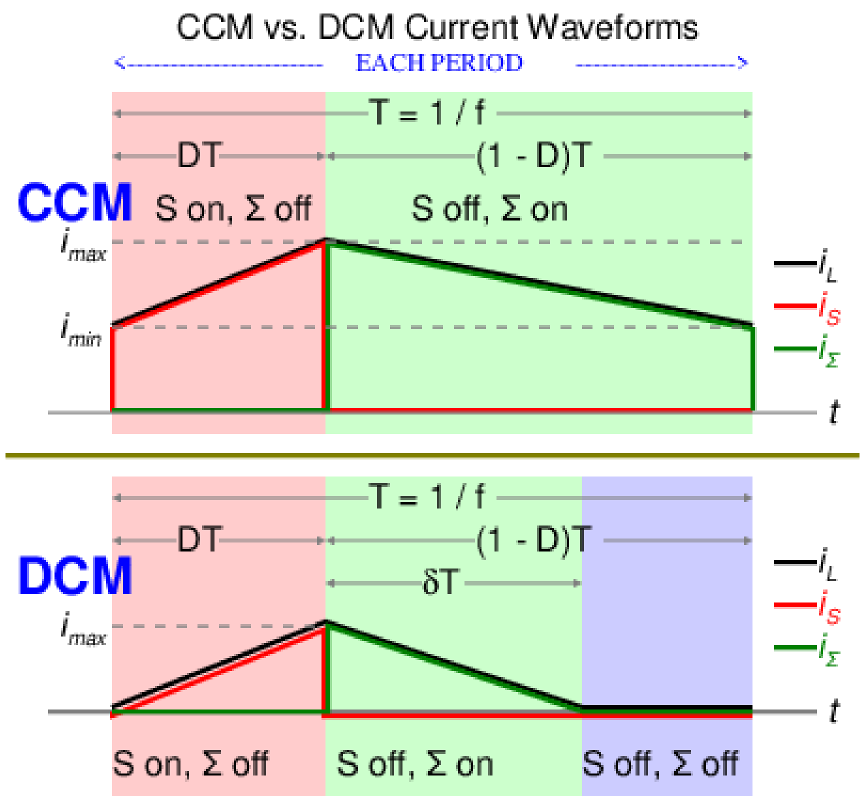

Figure 4: Comparison of CCM and DCM current waveforms.

The load is usually specified in terms of the current it takes. This leads to the simplest analysis of the converter. Once this is done, it is also possible to investigate what happens when a resistive load is connected.

The non-inverting buck-boost converter, shown in Fig. 5, provides a positive output voltage that could be smaller or greater than the input voltage. This converter requires two active switches and two diodes.

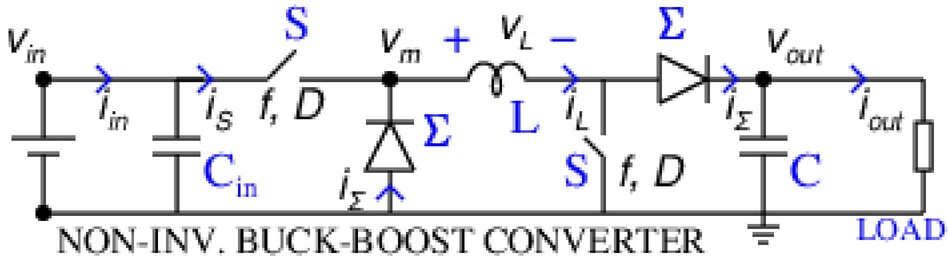

Figure 5: Non-inverting buck-boost converter.

2 Charging and discharging of inductors by a constant voltage

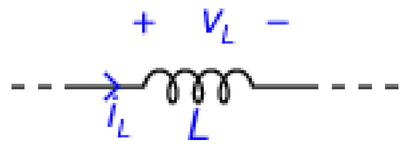

Figure 6: Inductor with voltage and current indicated.

For the inductor shown in Fig 6,

$$\frac{\mathrm{d}\, i_L}{\mathrm{d}\, t} = \frac{v_L}{L}. \tag{1}$$

If v_L is constant during an interval, then $\frac{\mathrm{d}\, i_L}{\mathrm{d}\, t}$ is also constant. Positive v_L causes i_L to rise with time and the inductor is said to be *charging*. Negative v_L causes i_L to fall with time and the inductor is said to be *discharging*. If v_L is constant during an interval, the change is current is simply $\frac{v_L}{L}$ times the length of the interval.

$$\Delta i_L = \frac{v_L}{L}\Delta t. \tag{2}$$

This equation will be used frequently for deriving expressions for the changes in the inductor current in switched mode power converters.

Note: $v_L \Delta t$ is the *volt-seconds* for this case.

3 Ripple voltage across a capacitor

Figure 7 shows grounded capacitor into which a periodic, zero-mean current i_C is flowing. For the present discussion, it is sufficient to consider the case in which the current changes sign only twice in a period. When i_C is positive, charge is being added to the capacitor. When i_C is negative, charge is taken out of the capacitor. As shown in the diagram, let the area under the positive part of the i_C curve be the charge

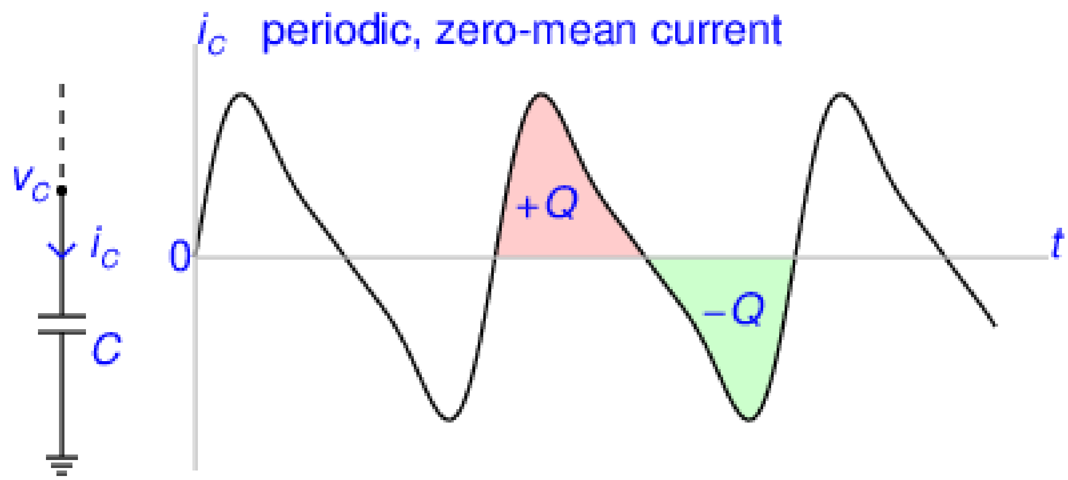

Figure 7: Grounded capacitor carrying a periodic, zero-mean current.

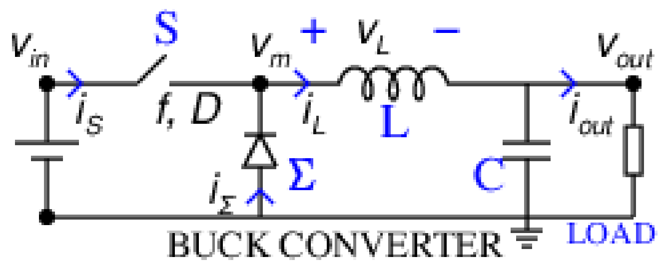

Figure 8: Buck converter circuit diagram

$+Q$. The same charge must be extracted from the capacitor when i_C is negative. This periodic addition and subtraction of the charge causes a periodic fluctuation in v_C, the voltage across capacitor. The voltage ripple, that is the maximum v_C minus the minimum v_C, is given by

$$(\text{Ripple}) = \frac{Q}{C}. \tag{3}$$

4 Buck converter

Figure 8 shows a buck converter. The output capacitance C is assumed to be large so that the output voltage v_{out} is nearly constant. In all derivations, it is first regarded as a constant. A formula for the ripple in the output voltage may sometimes be derived. The switch S operates at switching frequency f with duty factor D. The period is

$$T = \frac{1}{f}. \tag{4}$$

In each period, switch S remains on for duration

$$T_{\text{on}} = DT, \tag{5}$$

and off for duration

$$T_{\text{off}} = (1 - D)T. \tag{6}$$

When S is on, $v_m = v_{\text{in}}$. This positive voltage reverse biases the diode Σ and so Σ is off when S is on. When S is off, the inductor current i_L gets discharged through Σ. Two distinct modes of operation are possible.

In CCM (continuous conduction mode), the inductor current remains always positive and so Σ is on for the entire off duration $(1-D)T$ of S. In CCM, either S or Σ is on, S on for duration DT, Σ on for duration $(1-D)T$. There is no time during which both S and Σ are off. CCM waveforms are shown in Fig. 9.

In DCM (discontinuous conduction mode), the inductor current discharges to zero before the end of the off duration for S. Then Σ is on for duration δT, with $\delta < 1 - D$. To begin with, δ is not known and needs to be calculated using a formula that will be derived. In DCM, there is a duration in which *both S and Σ are off* before S turns on again. DCM waveforms are shown in Fig. 11.

4.1 Buck converter in CCM

In CCM, the inductor current is always positive. Fig. 9 shows all important waveforms associated with this mode of operation.

When switch S is on and Σ is off, $v_m = v_{\text{in}}$. Then $v_L = v_m - v_{\text{out}} = v_{\text{in}} - v_{\text{out}}$. During this interval, the change in current

$$i_{\max} - i_{\min} = \frac{v_{\text{in}} - v_{\text{out}}}{L} DT. \tag{7}$$

When switch S is off and Σ is on, $v_m = 0$. Then $v_L = v_m - v_{\text{out}} = -v_{\text{out}}$.

$$i_{\min} - i_{\max} = \frac{-v_{\text{out}}}{L}(1-D)T.$$

Or,

$$i_{\max} - i_{\min} = \frac{v_{\text{out}}}{L}(1-D)T. \tag{8}$$

Since the left hand sides of Eq.7 and Eq.8 are equal, the right hand sides are also equal.

$$\frac{v_{\text{in}} - v_{\text{out}}}{L} DT = \frac{v_{\text{out}}}{L}(1-D)T.$$

Or, $(v_{\text{in}} - v_{\text{out}})D = v_{\text{out}}(1-D)$. On simplification, this becomes

$$v_{\text{out}} = Dv_{\text{in}}. \tag{9}$$

From Fig. 8 it is seen that i_{L} is smoothed by capacitor C to give i_{out}. The average value of i_{L} is therefore i_{out}. From Fig. 9 it is clear that the average value of i_{L} is $(i_{\max} + i_{\min})/2$. So

$$i_{\max} + i_{\min} = 2i_{\text{out}}. \tag{10}$$

In Eq. 8, if v_{out} is substituted as Dv_{in}, and T as $1/f$, one obtains

$$i_{\max} - i_{\min} = D(1-D)\frac{v_{\text{in}}}{fL} = D(1-D)i_{\text{std}}. \tag{11}$$

Note that $v_{\text{in}}/(fL)$ is called i_{std}, the *standard current*.

$$i_{\text{std}} = \frac{v_{\text{in}}}{fL}. \tag{12}$$

From Eq. 10 and Eq. 11 one obtains

$$i_{\mathrm{max}} = i_{\mathrm{out}} + \frac{1}{2}D(1-D)i_{\mathrm{std}}, \tag{13}$$

and

$$i_{\mathrm{min}} = i_{\mathrm{out}} - \frac{1}{2}D(1-D)i_{\mathrm{std}}. \tag{14}$$

For CCM to be valid, i_{min} should be nonnegative. This requires

$$i_{\mathrm{out}} \geq \frac{1}{2}D(1-D)i_{\mathrm{std}}. \tag{15}$$

Let the normalized output current x be defined as

$$x = \frac{i_{\mathrm{out}}}{i_{\mathrm{std}}}. \tag{16}$$

Then in terms of x, the condition for CCM is

$$x \geq \frac{1}{2}D(1-D). \tag{17}$$

Let the normalized output voltage y be defined as

$$y = \frac{v_{\mathrm{out}}}{v_{\mathrm{in}}}. \tag{18}$$

In CCM, y is given by

$$y = D. \tag{19}$$

In CCM, the output voltage is independent of the current.

The input current is same as the switch current i_{S}. Its average value is seen from Fig. 9 to be $D(i_{\mathrm{max}} + i_{\mathrm{min}})/2 = Di_{\mathrm{out}}$. So the average input power is $v_{\mathrm{in}}Di_{\mathrm{out}} = Dv_{\mathrm{in}}i_{\mathrm{out}} = v_{\mathrm{out}}i_{\mathrm{out}}$, which is the output power. This verifies that the buck converter made from ideal components is lossless.

Eq. 11 gives the current ripple in the inductor. To estimate the ripple in the output voltage, Eq. 3 is used. Fig. 10 shows that Q is $\frac{1}{2}\frac{T}{2}\frac{i_{\mathrm{max}} - i_{\mathrm{min}}}{2} = \frac{T(i_{\mathrm{max}} - i_{\mathrm{min}})}{8} = \frac{TD(1-D)v_{\mathrm{in}}}{8fL} = \frac{D(1-D)v_{\mathrm{in}}}{8f^2L}$. Therefore, the ripple voltage is

$$v_{\mathrm{ripple,buck,CCM}} = \frac{Q}{C} = \frac{D(1-D)v_{\mathrm{in}}}{8f^2LC}. \tag{20}$$

4.2 Buck converter in DCM

In DCM, the inductor current falls to zero before the end of the period. Fig. 11 shows all important waveforms associated with this mode of operation. Here the current is 0 at the beginning of the period and increases to i_{max} during the time DT that S is ON. When S is switched OFF, the current continues to flow through Σ. It diminishes to 0 in time δT. After that both S and Σ are OFF.

Note that $\delta < 1 - D$. So the inductor discharges completely before the end of the S OFF duration. Why is the discharge so quick? This is because the v_{out} is now so high that it drains the current faster. The analysis that follows will demonstrate this.

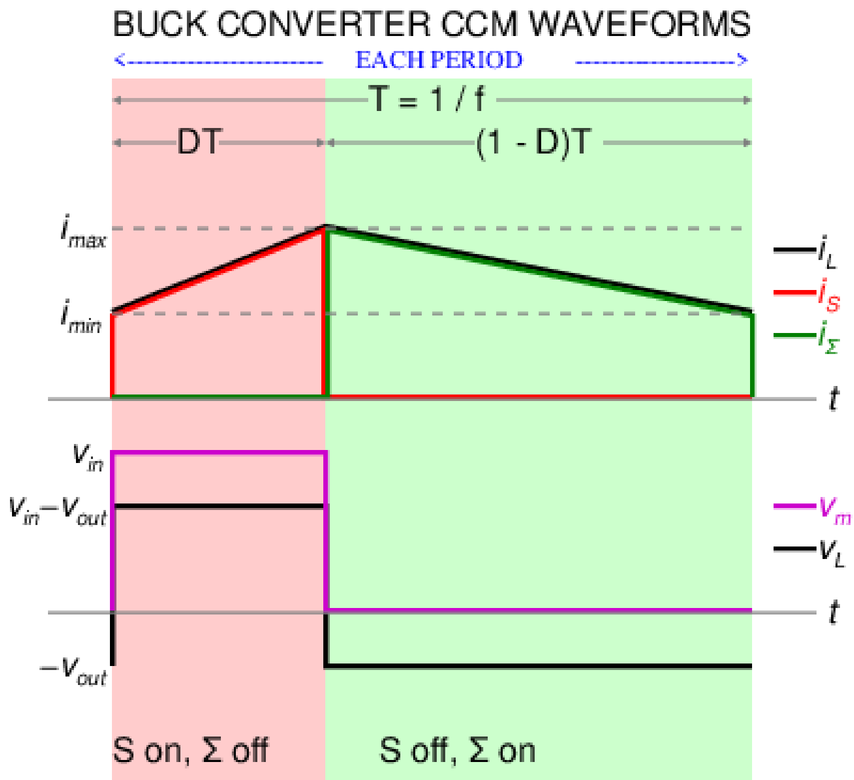

Figure 9: Buck converter CCM waveforms

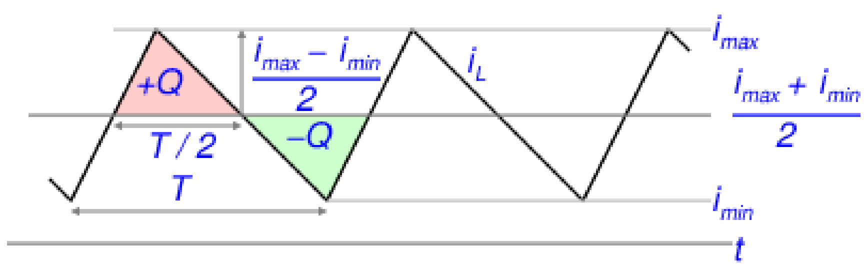

Figure 10: Charge for estimating the ripple voltage of a buck converter operating in CCM.

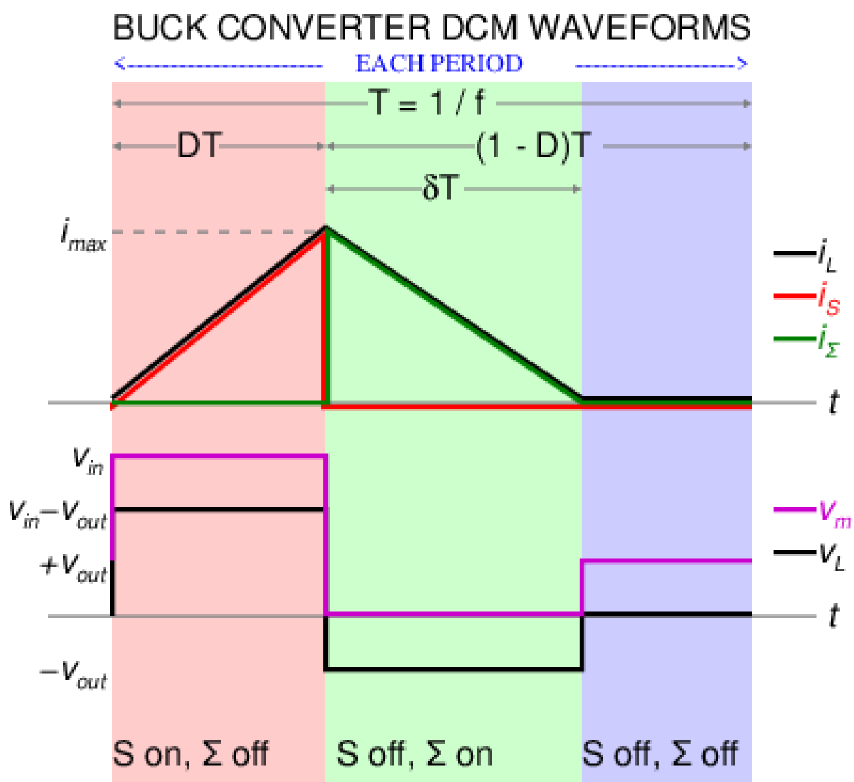

Figure 11: Buck converter DCM waveforms

When switch S is on and Σ is off, $v_m = v_{\text{in}}$. Then $v_L = v_m - v_{\text{out}} = v_{\text{in}} - v_{\text{out}}$. During this interval, the change in current

$$i_{\max} = \frac{v_{\text{in}} - v_{\text{out}}}{L} DT. \tag{21}$$

When switch S is off and Σ is on, $v_m = 0$. Then $v_L = v_m - v_{\text{out}} = -v_{\text{out}}$. During this interval, the change in current is

$$-i_{\max} = \frac{-v_{\text{out}}}{L} \delta T.$$

Or,

$$i_{\max} = \frac{v_{\text{out}}}{L} \delta T. \tag{22}$$

Since the left hand sides of Eq.21 and Eq.22 are equal, the right hand sides are also equal.

$$\frac{v_{\text{in}} - v_{\text{out}}}{L} DT = \frac{v_{\text{out}}}{L} \delta T.$$

Or, $(v_{\text{in}} - v_{\text{out}})D = v_{\text{out}}\delta$. On simplification, this becomes

$$v_{\text{out}} = \frac{D}{D + \delta} v_{\text{in}}. \tag{23}$$

Note that δ is NOT yet known.

From Fig. 8 it is seen that i_{L} is smoothed by capacitor C to give i_{out}. The average value of i_{L} is therefore i_{out}. From Fig. 11 it is clear that the average value of i_{L} is $(D + \delta)i_{\max}/2$. So

$$i_{\max} = \frac{2}{D + \delta} i_{\text{out}}. \tag{24}$$

In Eq. 22, if v_{out} is substituted as $\dfrac{D}{D+\delta} v_{\text{in}}$, and T as $1/f$, one obtains

$$i_{\max} = \frac{D\delta}{D + \delta} \frac{v_{\text{in}}}{fL} = \frac{D\delta}{D + \delta} i_{\text{std}}. \tag{25}$$

Note that $v_{\text{in}}/(fL)$ is i_{std}, the *standard current.*

The left hand sides of Eq. 25 and Eq. 24 are equal. So the right hand sides are also equal.

$$\frac{D\delta}{D + \delta} i_{\text{std}} = \frac{2}{D + \delta} i_{\text{out}}.$$

On simplification, an expression for δ in terms of the given parameters is obtained.

$$\delta = \frac{2}{D} \frac{i_{\text{out}}}{i_{\text{std}}} = \frac{2x}{D}. \tag{26}$$

Once δ is calculated, it can used in Eq. 23 to compute v_{out}, and in Eq. 24 to compute $i_{\max}$. This completes the determination of the output voltage and the inductor current for the DCM case of the buck converter.

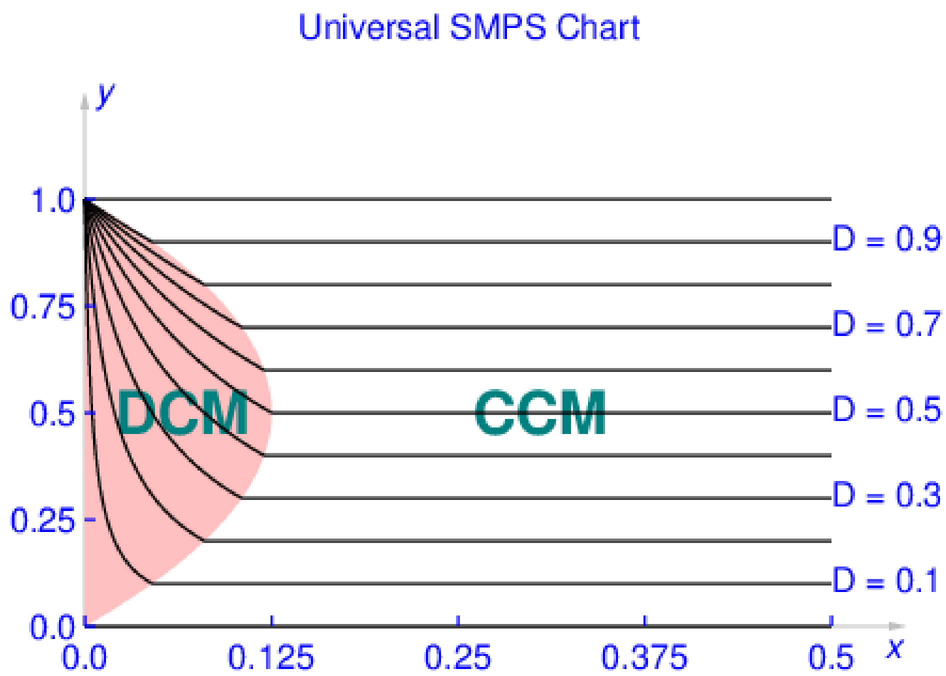

Figure 12: Buck converter chart.

4.3 Summary of buck converter calculations

Given parameters: v_{in}, f, D, L, and i_{out}

Tasks:

- Determine if the buck converter is in CCM or in DCM.
- CCM: Calculate v_{out}, i_{max}, and i_{min}.
- DCM: Calculate δ, v_{out}, and i_{max}.

Here is an outline of the procedure for these calculations.

- First compute $i_{\text{std}} = v_{\text{in}}/(fL)$.
- Then compute the normalized output current $x = i_{\text{out}}/i_{\text{std}}$.
- If $x < \frac{1}{2}D(1-D)$, then the buck converter is in **DCM**.
  - Compute $\delta = \dfrac{2x}{D}$.
  - Compute $v_{\text{out}} = \dfrac{D}{D+\delta}v_{\text{in}}$.
  - Compute $i_{\text{max}} = \dfrac{2}{D+\delta}i_{\text{out}}$.
- If $x \geq \frac{1}{2}D(1-D)$, then the buck converter is in **CCM**.
  - Compute $v_{\text{out}} = Dv_{\text{in}}$.

– Compute $i_{\max} = i_{\text{out}} + \dfrac{1}{2}D(1-D)i_{\text{std}}$.

– Compute $i_{\min} = i_{\text{out}} - \dfrac{1}{2}D(1-D)i_{\text{std}}$.

Chart: Let $y = v_{\text{out}}/v_{\text{in}}$. Then according to the analysis done so far

$$y = \begin{cases} D & x \geq \frac{1}{2}D(1-D) \\ \frac{D}{D+2x/D} & x < \frac{1}{2}D(1-D). \end{cases} \tag{27}$$

As y is expressed as a function of x, it is possible to show it in the chart of Fig. 12. Although this chart is for the buck converter, it will be seen later that calculations for the boost and the buck-boost converters can also be done using it.

4.4 Buck converter with a resistive load

If the load if specified not in terms of the current it takes, but as a resistance R, then the calculations become a little more involved.

What R values result in CCM? For CCM,

$$\frac{i_{\text{out}}}{i_{\text{std}}} = x \geq \frac{1}{2}D(1-D).$$

Or,

$$\frac{v_{\text{out}}/R}{v_{\text{in}}/(fL)} \geq \frac{1}{2}D(1-D).$$

But in CCM $v_{\text{out}}/v_{\text{in}} = D$. So

$$\frac{DfL}{R} \geq \frac{1}{2}D(1-D).$$

Or, the condition for CCM is

$$\frac{2fL}{R} \geq 1 - D. \tag{28}$$

Another way of writing this condition for CCM is

$$R \leq R_{\text{crit,buck}} = \frac{2fL}{1-D}. \tag{29}$$

What happens when $R > R_{\text{crit,buck}}$? Then the converter is in DCM. To calculate δ, one uses

$$\delta = \frac{2x}{D} = \frac{2}{D}\frac{i_{\text{out}}}{i_{\text{std}}} = \frac{2}{D}\frac{v_{\text{out}}/R}{v_{\text{in}}/(fL)} = \frac{2fL}{DR}\frac{v_{\text{out}}}{v_{\text{in}}} = \frac{2fL}{DR}\frac{D}{D+\delta} = \frac{2fL}{R}\frac{1}{D+\delta}.$$

Rearranging this, a quadratic equation in δ is obtained.

$$\delta^2 + D\delta - \frac{2fL}{R} = 0. \tag{30}$$

The sensible (positive) root is

$$\delta = -\frac{D}{2} + \sqrt{\left(\frac{D}{2}\right)^2 + \frac{2fL}{R}}. \tag{31}$$

Once δ is calculated, it can used in Eq. 23 to compute v_{out}. Then $i_{\text{out}} = v_{\text{out}}/R$, and Eq. 24 can be used to compute $i_{\max}$.