

PAPER

Quantum walk-based protocol for secure communication between any two directly connected nodes on a network

To cite this article: Prateek Chawla *et al* 2023 *Phys. Scr.* **98** 105113

View the [article online](#) for updates and enhancements.

You may also like

- [Malicious vehicle node detection mechanism based on repeated game and trust evaluation in VANET](#)
Yatao Zhu and Yongli Wang
- [Computational lymphatic node models in pediatric and adult hybrid phantoms for radiation dosimetry](#)
Choonsik Lee, Stephanie Lamart and Brian E Moroz
- [Analytical results for the distribution of first-passage times of random walks on random regular graphs](#)
Ido Tishby, Ofer Biham and Eytan Katzav



PAPER

Quantum walk-based protocol for secure communication between any two directly connected nodes on a network

RECEIVED
3 July 2023REVISED
7 September 2023ACCEPTED FOR PUBLICATION
18 September 2023PUBLISHED
28 September 2023Prateek Chawla^{1,2,*} , Adithi Ajith³ and C M Chandrashekar^{1,2,3} ¹ The Institute of Mathematical Sciences, C.I.T. Campus, Taramani, Chennai—600113, India² Homi Bhabha National Institute, Training School Complex, Anushakti Nagar, Mumbai 400094, India³ Quantum Optics & Quantum Information, Department of Instrumentation & Applied Physics, Indian Institute of Science, CV Raman Road, Bengaluru, Karnataka 560 012, India

* Author to whom any correspondence should be addressed.

E-mail: prateekc@imsc.res.in and chandru@imsc.res.in

Keywords: quantum walk, quantum network, quantum algorithm, quantum communication

Abstract

The utilization of quantum entanglement as a cryptographic resource has superseded conventional approaches to secure communication. Security and fidelity of intranetwork communication between quantum devices is the backbone of a quantum network. This work presents an protocol that generates entanglement between any two directly connected nodes of a quantum network to be used as a resource to enable quantum communication across that pair in the network. The protocol is based on a directed discrete-time quantum walk and paves the way for private inter-node quantum communication channels in the network. We also present the simulation results of this protocol on random networks generated from various models. We show that after implementation, the probability of the walker being at all nodes other than the source and target is negligible and this holds independent of the random graph generation model. This constitutes a viable method for the practical realisation of secure communication over any random network topology.

1. Introduction

A quantum network consists of a set of distributed quantum processors connected by quantum channels [1]. The quantum processors (nodes) are used for information processing tasks and the communication channels enable the transfer of quantum information between nodes. This enables the network to be a scalable solution for both quantum computation with a high number of qubits, and quantum communication networks over a large area [2]. This is a generalization of the classical models of distributed computing and communication [3–5]. Quantum clusters for distributed computing have the potential of providing a method to significantly improve the data processing capabilities of existing systems with only a linear increase in the resources (i.e., devices) required to realise the network [6, 7]. Protocols intended for implementation of distributed quantum computing are an active area of research [8–11], and the simulation of quantum networks and distributed protocols [12–14] have also attracted significant interest from the research community in recent times. Quantum networks to enhance communication have also been proposed and demonstrated. One of the most accessible technologies in this regard are the quantum key distribution (QKD) protocols to ensure secure communication [15–17]. The QKD networks have been deployed in large metropolitan settings [18–22], and have also been operationally demonstrated in networks connecting ground stations using satellites as trusted nodes [23–27], highlighting the utility of this approach.

One of the methods to implement various network-based protocols is to use the toolkit of the quantum walk formalism. Quantum walks on networks have been used for various applications such as search problems [28–31], state transfer and quantum routing [32–35], evaluation of information flow through networks [36–39], training of neural networks [40, 41], properties of percolation graphs [42–44], and universal quantum computation [45–48].

Quantum walks are a quantum generalization of a classical random walk. A major distinguishing feature between the two processes is that the quantum walk does not have any randomness associated with the dynamics, unlike a classical random walk. The randomness in the output of a quantum walk stems from the measurement-induced collapse of the walker's wavefunction [49]. Two of the well-studied variants of a quantum walk are the discrete-time and continuous-time quantum walks. The continuous-time variant is described using only the position Hilbert space of the walker, whereas, the discrete-time variant requires an additional internal Hilbert space, dubbed the coin space of the walker. Continuous-time formalism, for example, has been effectively used in spatial search protocols [50], in defining graph kernels [51], encryption algorithms [52], and in modelling of energy transfer in photosynthesis [53]. The discrete-time quantum walk (DTQW) formalism offers the possibility of engineering the dynamics of the walker with more control, due to an additional degree of freedom provided by the coin Hilbert space. Along with its use in search protocols [30, 54–56], it has been used to model topological phenomena [57–61], dynamics of Dirac cellular automata [62–67], neutrino oscillations [68], among others.

Consider a quantum particle in a Hilbert space \mathcal{H} , defined as $\mathcal{H} = \mathcal{H}_w \otimes \mathcal{H}_i$, where \mathcal{H}_w is the Hilbert space in which the quantum walk takes place, and the state in \mathcal{H}_i encodes the information to be transported. Since both the state undergoing the quantum walk and the information-carrying state belong to different Hilbert spaces, they are encoded in two different degrees of freedom of the particle. The information-carrying state is only accessible via measurement if the particle is detected at the specific node where the measurement is made. In this manuscript, we focus on the quantum walk protocol, as the transport of information will be trivially successful after its implementation.

We propose a protocol that makes use of a directed variant of the discrete-time quantum walk on a network to create an entangled state between any two connected nodes of the network. We show that this protocol results in the walker being found with a high probability at either the source or the target nodes, and with a negligibly small chance of being found at any other node. This result is demonstrated over random networks generated by a few different models used to generate networks that share characteristics with some real-world large-scale networks. This highlights the versatility of our protocol and prompts its utility on quantum networks at various scales.

Since quantum walks have also been experimentally realized in several systems, [69–71] and the operations which we have used are all unitaries, it is indicative that the protocol proposed in this study is experimentally realizable.

This paper is organized as follows. In section 2.1, we outline the form of directed DTQW on a network, and we show the construction of the protocol in section 2.2. Further, section 2.3 describes a qualitative use of von Neumann entropy as a secondary confirmation of the working of the protocol. Section 3 showcases the results of applying our protocol for several different network topologies. We summarize our findings and conclude in section 4.

2. Quantum walk protocol

In our protocol, we attempt to create a state such that the probability of the particle to be found is maximized between two pre-selected nodes of a quantum network, and negligible everywhere else. The network is represented as a graph $\Gamma = (V, E)$, where V, E represent the sets of its vertices and edges, respectively. We make use of a quantum ratchet operator [72] in conjunction with a directed discrete-time quantum walk protocol to model the dynamics of the quantum particle on such a graph. We shall first describe the directed discrete-time quantum walk in section 2.1, and then use it to describe the protocol in section 2.2. A qualitative explanation of the results section 2.3.

2.1. Directed discrete-time quantum walk on a graph

The discrete-time evolution of a quantum walker on an infinite one-dimensional lattice is described on a Hilbert space which is isomorphic to that of a composite system of a qubit and a qudit. Mathematically, the Hilbert space is defined as $\mathcal{H}_w = \mathcal{H}_c \otimes \mathcal{H}_p$, where \mathcal{H}_c is the coin Hilbert space, and \mathcal{H}_p is the position Hilbert space of the walker. The evolution of the particle proceeds with the repeated application of quantum coin operation $C(\theta)$ acting only on the coin Hilbert space followed by the conditional shift operator S acting on the complete, coin and position Hilbert space \mathcal{H}_w . These operators are of the form,

$$C(\theta) = \begin{bmatrix} \cos(\theta) & i \sin(\theta) \\ i \sin(\theta) & \cos(\theta) \end{bmatrix}$$

$$S = \sum_{x \in \mathbb{Z}} [|\uparrow\rangle\langle\uparrow| \otimes |x-1\rangle\langle x| + |\downarrow\rangle\langle\downarrow| \otimes |x+1\rangle\langle x|], \quad (1)$$

where the set $\{|\uparrow\rangle, |\downarrow\rangle\}$ is chosen to represent the orthonormal basis of \mathcal{H}_c and the elements of $\{|x\rangle, \forall x \in \mathbb{Z}\}$ label the eigenstates of \mathcal{H}_p . This formulation is easily modified to adjust for lattices of finite dimension. In full generality, the operator $C(\theta)$ is a 3-parameter $SU(2)$ rotation matrix, however, we choose the convention of using a 1-parameter form, fixing the other two parameters to be 0 and $\frac{3\pi}{2}$ to obtain the form shown in equation (1).

The evolution of the quantum walker without loss of generality may be considered to begin from a localized position eigenstate and a randomly oriented vector in the coin Hilbert space. The dynamical equation of evolution is then given by,

$$|\psi(t)\rangle = [S(C(\theta) \otimes \mathbb{1}_p)]^t |\psi(0)\rangle, \quad (2a)$$

where,

$$|\psi(0)\rangle = (\alpha|\uparrow\rangle + \beta|\downarrow\rangle) \otimes |x=0\rangle. \quad (2b)$$

Here $\alpha, \beta \in \mathbb{C}$ are chosen such that the coin state is normalized, i.e. $|\alpha|^2 + |\beta|^2 = 1$, and $\mathbb{1}_p$ represents the identity operation on the position Hilbert space. The discrete-time quantum walk is subject to many variations [73–75], and in this case, we consider the directed discrete-time quantum walk on a graph, as described in [38]. The (directed) shift operation is then defined as,

$$S = \sum_x \left[|\uparrow\rangle\langle\uparrow| \otimes |x\rangle\langle x| + \sum_j (|\downarrow\rangle\langle\downarrow| \otimes U_{jx|j}) \langle x| \right]. \quad (3)$$

Here, $U = e^{iL}$, where L is Laplacian of the graph, defined by its matrix elements L_{pq} , given by

$$L_{pq} := \begin{cases} \deg(v_p) & p = q \\ -1 & (p, q) \in E \\ 0 & (p, q) \notin E \end{cases} \quad (4)$$

where $\deg(v_p)$ is the degree of $v_p \in V$. The Laplacian of a graph is also given as $L = \gamma(D - A)$, where $\gamma \in \mathbb{R}$, D is known as the degree matrix, and A is the adjacency matrix of the graph. This form of the shift operator ensures that the walker may only walk along an edge that exists and may not jump to an unconnected node. This helps to restrict the evolution of the walker in the position space to that allowed by the network structure. The quantum coin is implemented using a ratchet formalism [72], where the source may choose a destination node for state transportation, and the target may switch between two different values of the coin operator. Let $W = \{s, t\}$ be a set containing the source and target nodes, labelled by the basis vectors $|s\rangle$ and $|t\rangle$, respectively, of \mathcal{H}_p . Assuming the scenario of only one-to-one communication, the node-dependent coin operator may be defined as,

$$C_{rat}(V, W) = \sum_{v \in V \setminus W} C\left(\frac{\pi}{2}\right) \otimes |v\rangle\langle v| + \sum_{w \in W} C(0) \otimes |w\rangle\langle w|. \quad (5)$$

2.2. Description of the protocol

The protocol for achieving state transport across the quantum network requires a preexisting networking infrastructure so that the source is able to identify the target without error. Additionally, we consider a weaker requirement for a secure classical communication system to communicate with the target node. This can later be extended into a fully quantum protocol using higher-dimensional quantum switches, which does not require the classical channel.

In our protocol, each node is able to choose the coin operator that it will implement locally, as per equation (5). By default, all nodes use the coin $C\left(\frac{\pi}{2}\right)$, as $W = \emptyset$, i.e. the source and target nodes are not yet defined. The source node is then identified and switches its coin operation to $C(0)$, signals the target node to do the same, and additionally, changes the value of the parameter k . In our simulations, we have set $k = 400$, but any $k \gtrsim \mathcal{O}(10^2)$ is acceptable for the protocol to work. Lower values result in higher losses. The walker then executes a directed discrete-time quantum walk, with the initial state being given by,

$$|\eta(0)\rangle = |\downarrow\rangle \otimes |s\rangle \quad (6)$$

following the evolution shown in equations (2a) and (2b), where the shift and coin operators are replaced by their directed and ratcheted counterparts described on networks, shown in equations (3) and (5), respectively. A summary of the protocol is shown in Prot. 1.

Protocol 1. Quantum walk protocol for transport on network.

Require: Adjacency matrix A for graph $\Gamma = (V, E)$.

Ensure: The source (s) and target (t) nodes exist.

Let set of vertices is V , and $W = \{s, t\}$.

$A_{st} \leftarrow kA_{st}$, where $k \gtrsim \mathcal{O}(10^2)$.

Set constant $\gamma \in \mathbb{R}$

Set evolution time $\tau \in \mathbb{Z}_+$

Set $L \leftarrow \gamma(D - A)$

procedure D-DTQWNetwork L, V, W, τ

Set initial state $|\psi(0)\rangle = |\downarrow\rangle \otimes |s\rangle$.

Set time counter $n = 0$

while $n < \tau$ **do**

Apply walk operation $|\psi(n+1)\rangle \leftarrow [SC_{rat}]|\psi(n)\rangle$

$n \leftarrow n + 1$

end while

return $|\psi(\tau)\rangle$

end procedure

Interestingly, it is known that quantum walks localize the walker in case of temporal and/or spatial disorder in the dynamics [61, 76–79]. Thus in case of an eavesdropper in the system, the effect of their presence directly translates to noise in quantum walk dynamics, which localizes the walker at the source. This ensures the security of this protocol, as in case of noise (i.e., eavesdroppers) in the network, the walker will localize at the source and never move at all.

2.3. Entanglement within the network

In order to create a scenario where the particle has a high probability of being found between only two position points, we consider the entanglement (measured via von Neumann entropy) between its position and coin Hilbert spaces, described in section 2.1 as \mathcal{H}_p and \mathcal{H}_c , respectively. Physically, this joint state may be viewed as representing a qubit local to each vector in the position eigenbasis. As the particle traverses this network (i.e., upon applications of the shift operation of equation (3)), the action of the coin operator (see equation (5)) may be seen as manipulating these qubits ‘local’ to each basis vector [80]. Thus the evolution of the ‘local’ coin state may be seen as,

$$\rho_c^{ii}(N) = \text{Tr}_p[(\mathbb{1}_c \otimes |i\rangle_p \langle i|) \rho(N)], \quad (7)$$

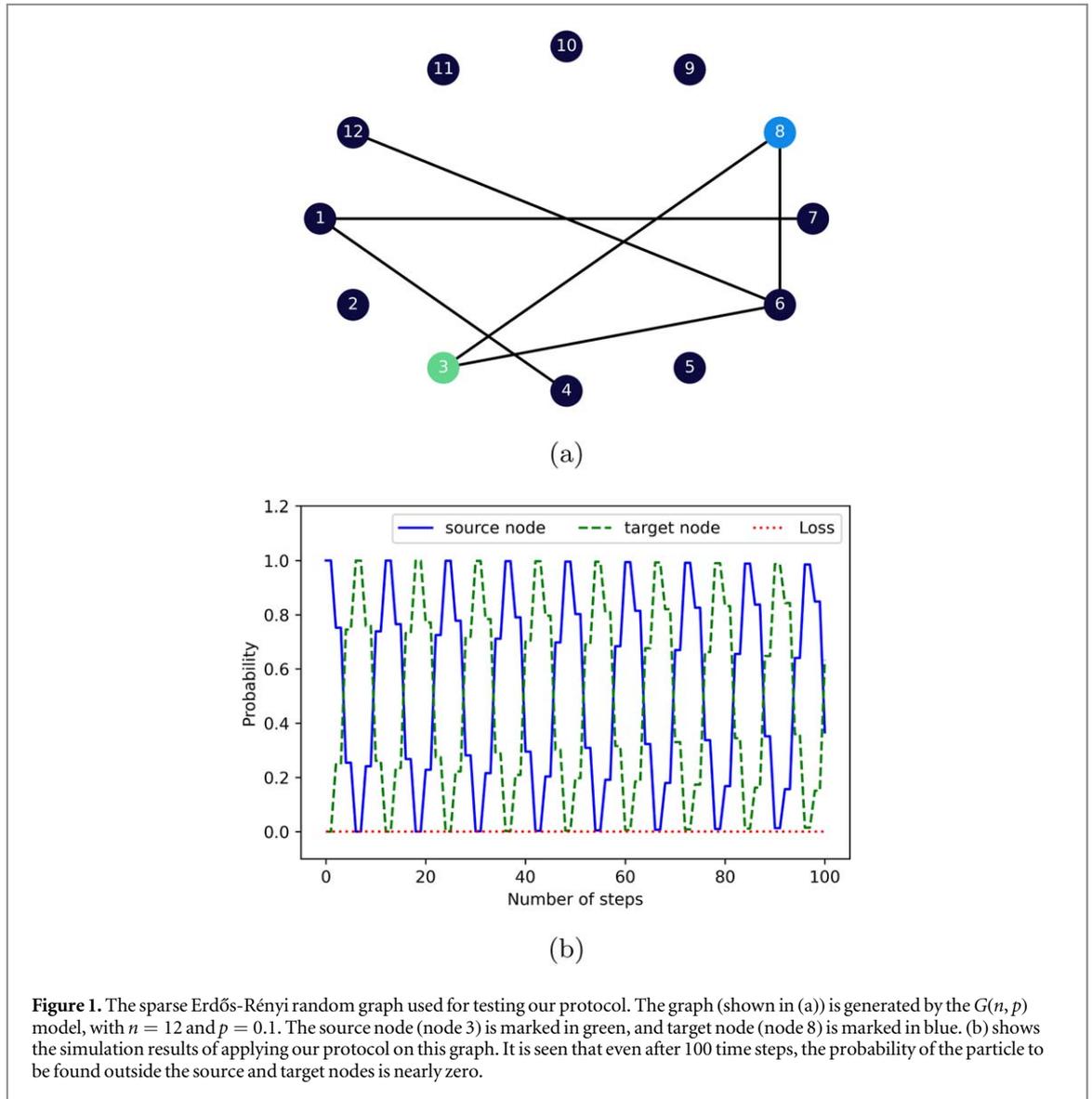
where $|i\rangle$ is an element of an orthonormal basis set of \mathcal{H}_p , $\rho(N)$ is the density matrix corresponding to the evolved state returned by the Prot. 1 after N steps of evolution. The $\rho_c^{ii}(N)$ is then the (unnormalized) reduced density matrix corresponding to the qubit corresponding to the basis vector $|i\rangle$ of the position space. The normalization is achieved by post selecting on the events when the particle wavefunction collapses to $|i\rangle_p$ upon the measurement in the position space. This interpretation may be extended further to include coherences between any two vectors of the orthonormal basis set, and one may construct a reduced joint density matrix of two such qubits local to the basis vectors $|i\rangle$ and $|j\rangle$. This is consistent with the tensor product interpretation, as upon extending this formulation to include the entire eigenbasis of \mathcal{H}_p (by considering the joint density matrices of states local to multiple basis vectors), one obtains the full density matrix $\rho(N)$ of the system. The construction of the reduced density matrix (following equation (7)) will then look like,

$$\tilde{\rho}_c^{ij}(N) = \begin{bmatrix} \rho_c^{ii}(N) & \rho_c^{ij}(N) \\ \rho_c^{ji}(N) & \rho_c^{jj}(N) \end{bmatrix}, \quad (8)$$

where $\rho_c^{ii}(N)$ is used in a generalized form given as,

$$\rho_c^{mn}(N) = \text{Tr}_p\{(\mathbb{1}_c \otimes |m\rangle_p \langle n|) \rho(N)\}, \quad m, n \in V, \quad (9)$$

where V is the set of nodes of the graph and $\tilde{\rho}_c^{ij}(N)$ is a reduced density matrix of a 2-qubit system. This enables one to evaluate measures of entanglement on this system, which is an indication of the existence of a local quantum channel between these qubits. This can be used as a qualitative indication for the existence of a local quantum channel within the network. In this manuscript, we use the von Neumann entropy as a measure of entanglement.



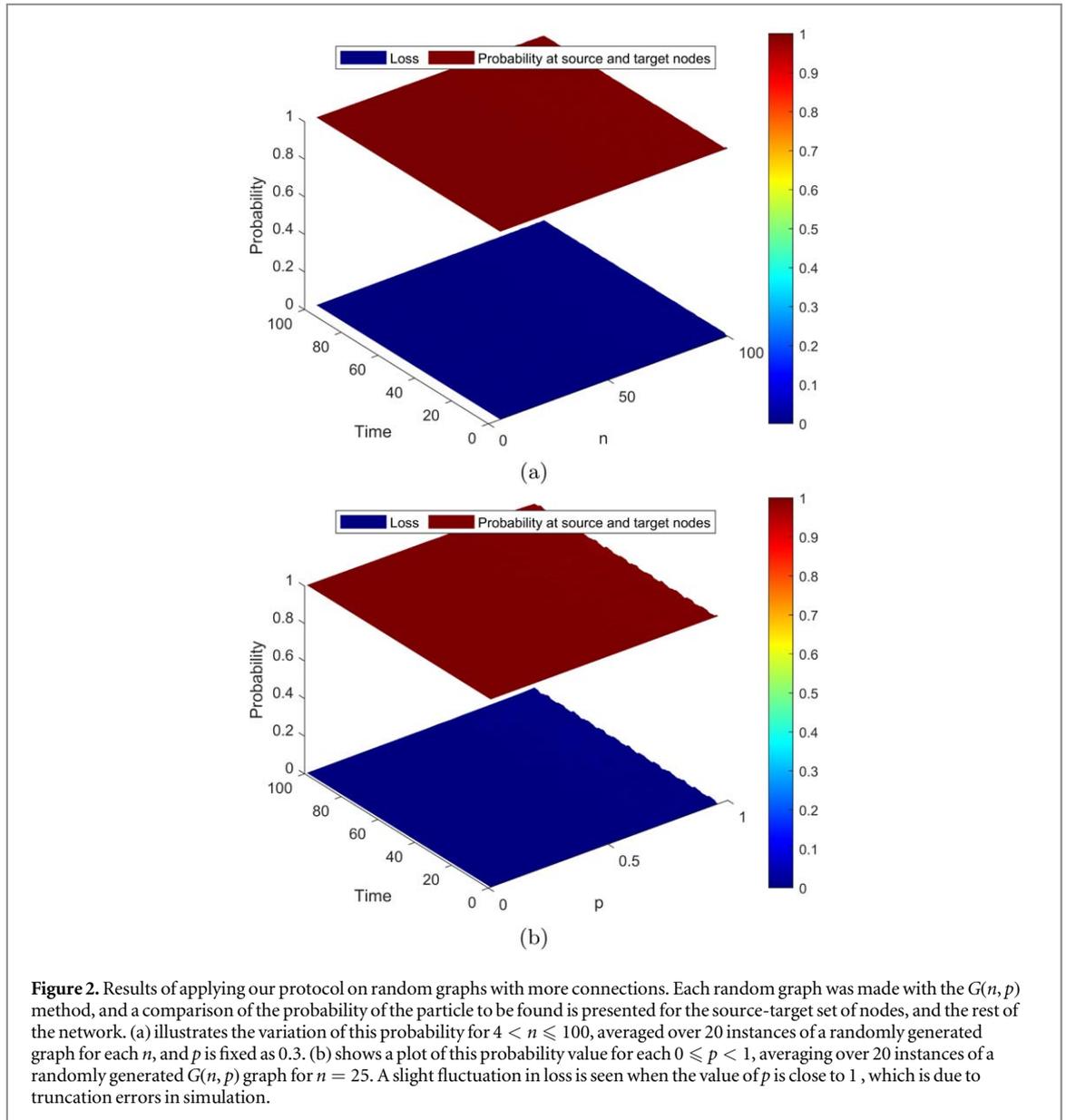
3. Results of simulation

3.1. Evolution of probability distribution with time

In this section, we present the results of the simulation on random graphs created by several methodologies. We first demonstrate this method on a sparse Erdős-Rényi random graph (also known as the $G(n, p)$ model), as shown in figure 1. In this case, we consider the probability of the particle to be detected at any node $v \in V W$ as a 'loss'. It is seen that the probability of the walker oscillates between the source and target nodes over time, without losses into the rest of the network. A similar behavior is seen when the number of connections in the random graph is increased, as in figure 2.

The protocol also shows similar behavior on random graphs created by other strategies, such as the NewmanWattsStrogatz (NWS) protocol [81]. This method generates a random graph by first constructing a ring with N nodes, then connecting the ring to its k nearest neighbours. For each node w in the N -ring, an edge (w, m) is added with probability p , for a randomly selected node m . This method has the advantage of creating clustering in the graph structure while retaining a short average path length. A simulation of our protocol on the NWS graph with $N = 34$, $k = 3$, and $p = 0.3$ is shown in figure 3.

The Erdős-Rényi model to generate random graphs can be seen as a snapshot of a stochastic process, which adds more nodes and edges to the network over time. This is useful for applications such as modelling bond percolation, but it creates a degree distribution which does not model real-world networks very well. Specifically, they do not feature a high clustering coefficient, and the degree distribution of their nodes does not approach a power law. This is somewhat accounted for by the use of the NWS protocol, which is able to account for the clustering behaviour. In order to achieve a power law degree distribution, other models have to be used.



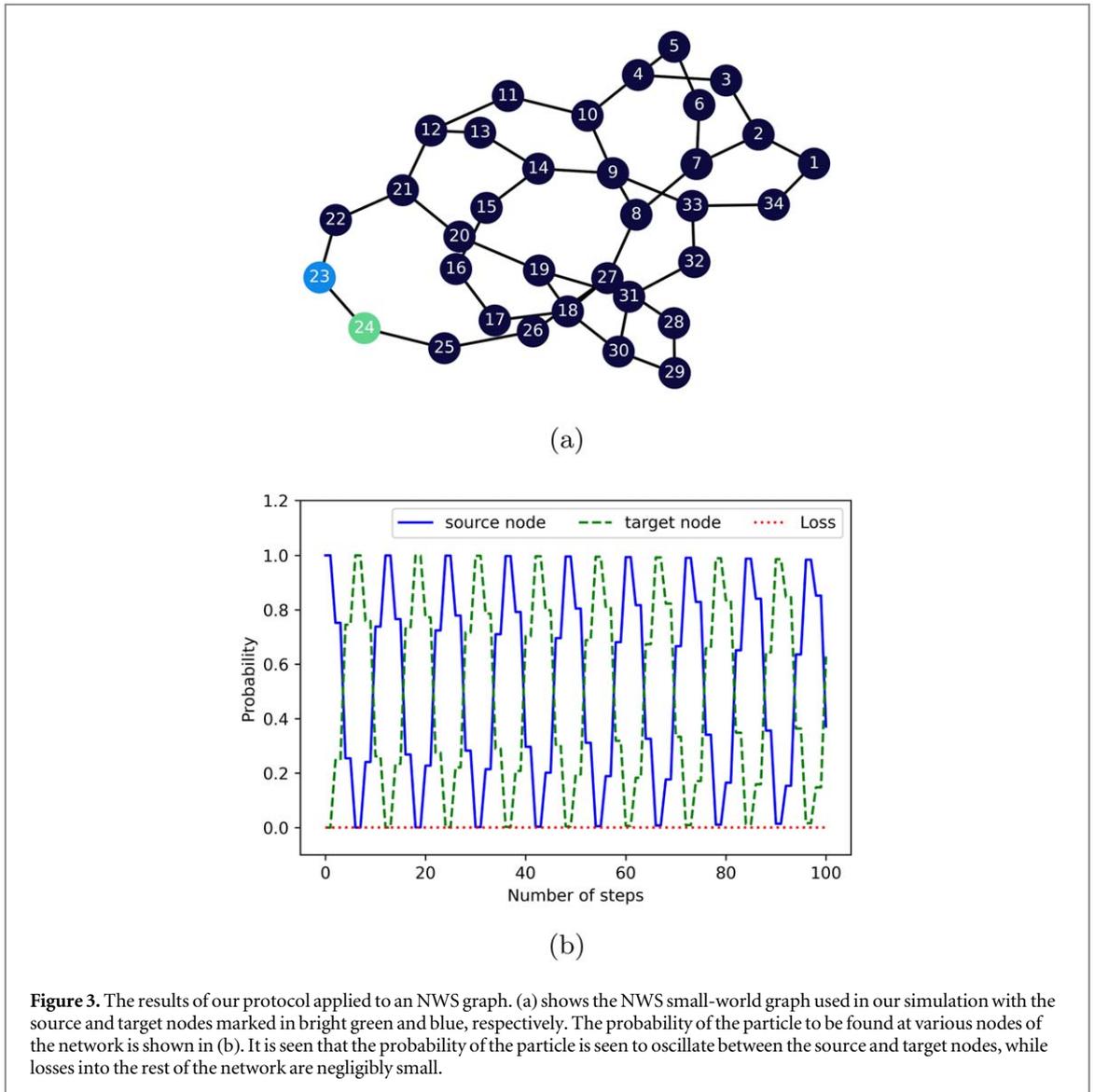
In this case, we demonstrate the protocol on graphs generated by the Barabási-Albert model [82]. This model supports features like growth, as well as preferential attachment, which is useful to emulate features observed in some real-world networks. Figure 4 shows a random graph generated by this model, as well as the results obtained by implementation of our protocol on this graph. It may be shown via simulation that the protocol is able to localize the walker between the source and target nodes for any such graph, independent of the generative parameters.

Thus, we see that irrespective of the number of connections in the random graph, or the method of graph generation, the probability of the particle oscillates between the position spaces of the source and target nodes with negligible losses to other nodes. This also underscores the security aspect of this protocol, as it localizes the particle between the source and target nodes, i.e. any interference by a third party can be detected as a loss of fidelity of the measured state of the particle.

3.2. Evolution of von Neumann entropy with time

We show the variation of the von Neumann entropy between the source and target nodes, as well as the target and a node randomly selected from the rest of the network in figure 5.

Thus this protocol is able to selectively create an entangled state between the local qubits of two selected (source and target) position basis vectors. In case the coin Hilbert space is traced out and only the probability of the particle to exist at a certain position is measured, then that curve (see figures 2, 3, and 4) shows oscillations between the source and target nodes.

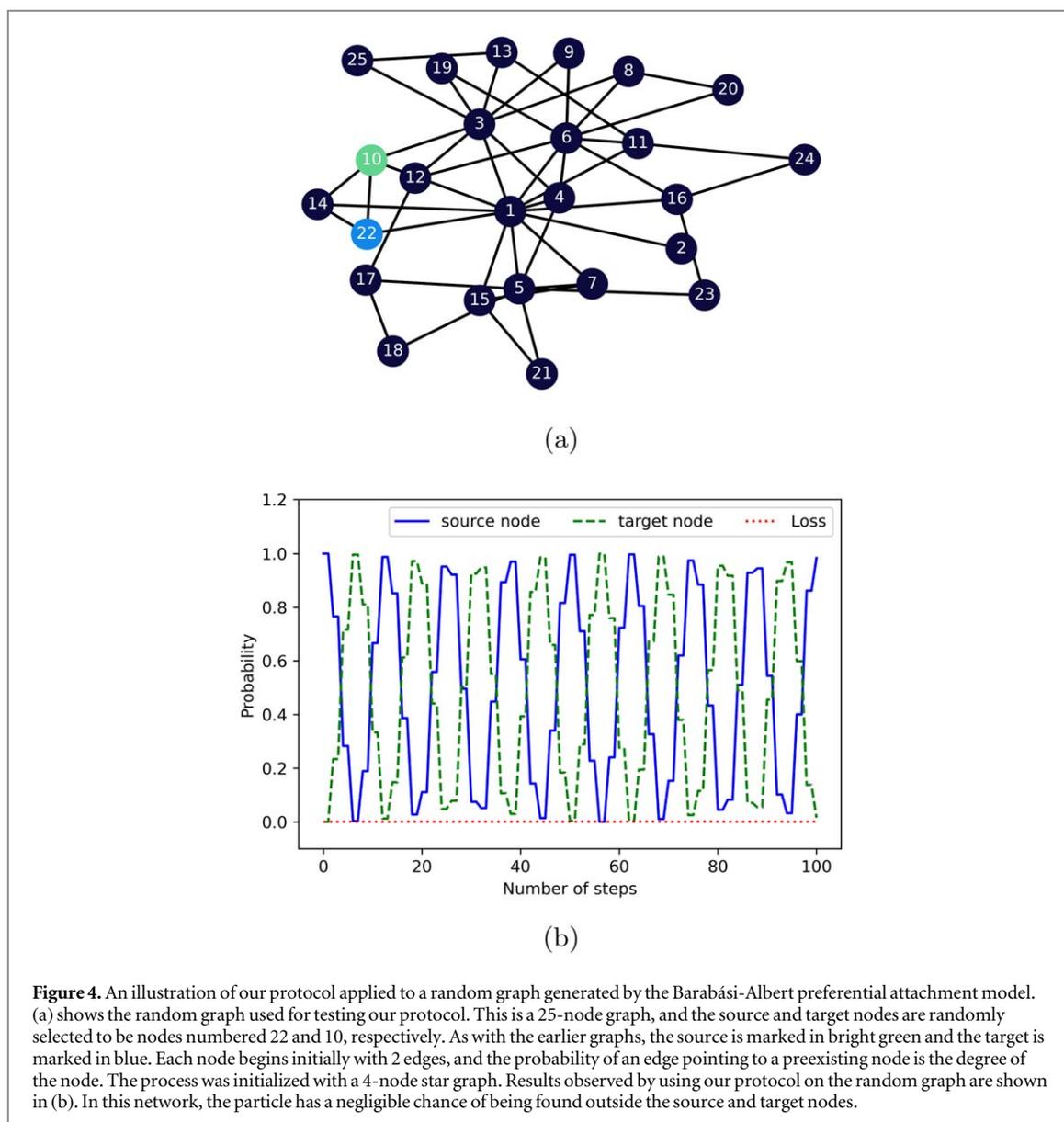


4. Conclusions

In this work, we have demonstrated an protocol that is capable of enabling secure communication between two specific nodes on a quantum network. The dynamics of a particle on the quantum network are modelled as a directed discrete-time quantum walk on a graph, where the structure of the network is captured by the adjacency matrix of the graph.

The dynamical behaviour of the particle is directed by the protocol such that it has a high probability of being found at either the source or the target nodes, with a negligibly small probability of being found at any other node. We test our protocol on Erdős-Rényi, Newman-Watts-Strogatz, and Barabási-Albert graphs, and show that it is able to produce the desired output independent of the method of graph generation. This indicates the potential utility of this protocol on real-world realizations of quantum networks at various scales.

This can contribute to the security of communication and transport operations across quantum networks. The requirement of a secure classical channel can be obviated if the source is able to access the state of a quantum switch, which can then be used to identify the target and change its coin operator. With suitable modifications, this protocol can be used for communication systems over any network topology and presents a promising model for the establishment of private, local quantum communication channels on existing networks. This model can be extended in the future, to also address cases where the source and target are connected with a path of length greater than 1.



Acknowledgments

P C would like to thank Tanmay Saha for the insightful discussions. We acknowledge support from the Interdisciplinary Cyber-Physical Systems (ICPS) Programme of the Department of Science and Technology, Government of India. Grant No. DST/ICPS/QuST/Theme-1/2019.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

Statements and declarations

Competing interests

The authors have no competing interests to declare that are relevant to the content of this article.

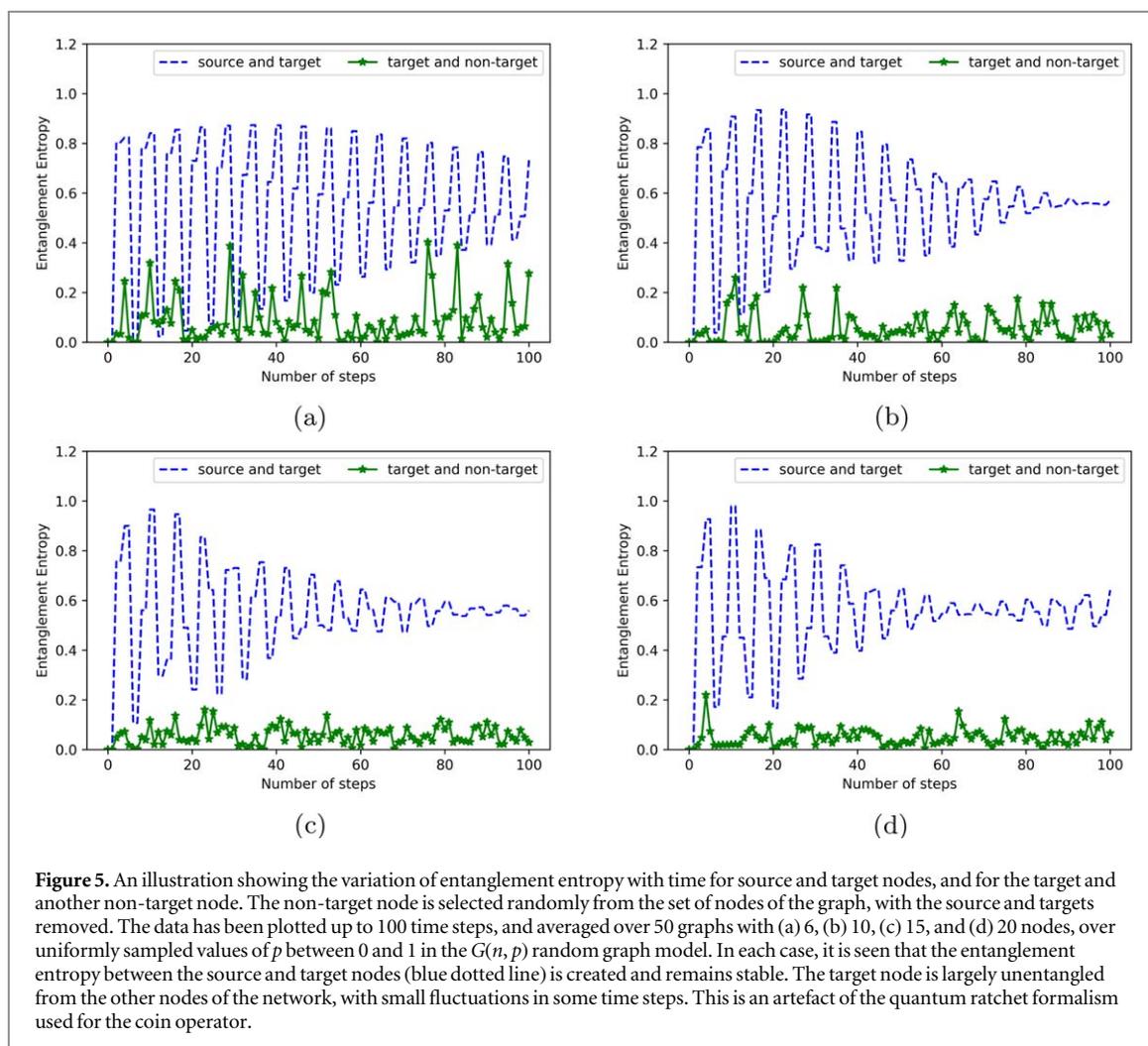


Figure 5. An illustration showing the variation of entanglement entropy with time for source and target nodes, and for the target and another non-target node. The non-target node is selected randomly from the set of nodes of the graph, with the source and targets removed. The data has been plotted up to 100 time steps, and averaged over 50 graphs with (a) 6, (b) 10, (c) 15, and (d) 20 nodes, over uniformly sampled values of p between 0 and 1 in the $G(n, p)$ random graph model. In each case, it is seen that the entanglement entropy between the source and target nodes (blue dotted line) is created and remains stable. The target node is largely unentangled from the other nodes of the network, with small fluctuations in some time steps. This is an artefact of the quantum ratchet formalism used for the coin operator.

ORCID iDs

Prateek Chawla  <https://orcid.org/0000-0001-8895-2791>

C M Chandrashekar  <https://orcid.org/0000-0003-4820-2317>

References

- [1] Kimble H J 2008 The quantum internet *Nature* **453** 1023
- [2] Duan L-M and Monroe C 2010 Colloquium: quantum networks with trapped ions *Rev. Mod. Phys.* **82** 1209
- [3] Tanenbaum A S and van Steen M 2002 *Distributed Systems: Principles and Paradigms* ed nachdr (Prentice Hall)
- [4] Kesidis G 2007 *An Introduction to Communication Network Analysis* (Wiley) (<https://doi.org/10.1002/9780470168684>)
- [5] Gries D and Schneider F B (ed) 2010 *Distributed Programs, in Verification of Sequential and Concurrent Programs* (Springer) 373–406
- [6] Caleffi M, Cacciapuoti A S and Bianchi G 2018 Quantum internet: from communication to distributed computing! *Proceedings of the 5th ACM International Conference on Nanoscale Computing and Communication, ACM* 1–4
- [7] Wehner S, Elkouss D and Hanson R 2018 Quantum internet: a vision for the road ahead *Science* **362** eaam9288
- [8] Yimsiriwattana A and Lomonaco S J Jr. 2004 Distributed quantum computing: a distributed Shor algorithm *Distributed quantum computing: a distributed Shor algorithm Defense and Security* 5436 ed E Donkor, A R Pirich and H E Brandt (Orlando, FL: SPIE) 360
- [9] Van Meter R 2014 *Quantum Networking* ed M Dias de Amorim (London, UK: Wiley- ISTE) 1st ed. Networks and Telecommunications Series 978-1-118-64893-3
- [10] Jin Z-X and Fei S-M 2019 Finer distribution of quantum correlations among multiqubit systems *Quantum Inf. Process.* **18** 21
- [11] Sundaram R G, Gupta H and Ramakrishnan C R 2022 Distribution of Quantum Circuits Over General Quantum Networks 2022 *IEEE International Conference on Quantum Computing and Engineering (QCE) (Broomfield, CO, USA, September 2022)* (Los Alamitos, CA, USA: IEEE Computer Society) 415–425
- [12] Bartlett B 2018 *arXiv* 10.48550/arXiv.1808.07047 21 Aug 2018[quant-ph]A distributed simulation framework for quantum networks and channels
- [13] Diadamo S, Notzel J, Zanger B and Bese M M 2021 QuNetSim: a software framework for quantum networks *IEEE Transactions on Quantum Engineering* **2** 1
- [14] Wu X, Kolar A, Chung J, Jin D, Kettimuthu R and Suchara M 2021 *arXiv* <https://doi.org/10.48550/arXiv.2111.03918> Nov 2021 [quant-ph]Parallel simulation of quantum networks with distributed quantum state management
- [15] Elliott C 2002 Building the quantum network* *New J. Phys.* **4** 46

- [16] Sasaki M *et al* 2011 Field test of quantum key distribution in the Tokyo QKD network *Opt. Express* **19** 10387
- [17] Lauritzen B, Minář J, de Riedmatten H, Afzelius M and Gisin N 2011 Approaches for a quantum memory at telecommunication wavelengths *Phys. Rev. A* **83** 012318
- [18] Poppe A, Peev M and Maurhart O 2008 Outline of the SECOQC quantum-key-distribution network in Vienna *International Journal of Quantum Information* **06** 209
- [19] Wang S *et al* 2014 Field and long-term demonstration of a wide area quantum key distribution network *Opt. Express* **22** 21739
- [20] Azuma K, Tamaki K and Munro W J 2015 All-photonics intercity quantum key distribution *Nat. Commun.* **6** 10171
- [21] Ou Y, Hugues-Salas E, Ntavou F, Wang R, Bi Y, Yan S, Kanellos G, Nejabati R and Simeonidou D 2018 Field-trial of machine learning-assisted quantum key distribution (QKD) networking with SDN *2018 European Conference on Optical Communication (ECOC), IEEE* 1–3
- [22] Dynes J F *et al* 2019 Cambridge quantum network *npj Quantum Information* **5** 101
- [23] Bedington R, Arrazola J M and Ling A 2017 Progress in satellite quantum key distribution *npj Quantum Information* **3** 30
- [24] Liao S-K *et al* 2018 Satellite-relayed intercontinental quantum network *Phys. Rev. Lett.* **120** 030501
- [25] Pan Z and Djordjevic I B 2020 Security of satellite-based CV-QKD under realistic assumptions *2020 22nd International Conference on Transparent Optical Networks (ICTON), IEEE (Bari, Italy, July 2020)* (IEEE) 1–4978-1-7281-8423-4
- [26] Chen Y-A *et al* 2021 An integrated space-to-ground quantum communication network over 4,600 kilometres *Nature* **589** 214
- [27] Li Y *et al* 2022 Space-ground QKD network based on a compact payload and medium-inclination orbit *Optica* **9** 933
- [28] Novo L, Chakraborty S, Mohseni M, Neven H and Omar Y 2015 Systematic dimensionality reduction for quantum walks: optimal spatial search and transport on non-regular graphs *Sci. Rep.* **5** 13304
- [29] Chakraborty S, Novo L, Ambainis A and Omar Y 2016 Spatial search by quantum walk is optimal for almost all graphs *Phys. Rev. Lett.* **116** 100501
- [30] Wong T G 2018 Faster search by lackadaisical quantum walk *Quantum Inf. Process.* **17** 68
- [31] Qu D, Marsh S, Wang K, Xiao L, Wang J and Xue P 2022 Deterministic search on star graphs via quantum walks *Phys. Rev. Lett.* **128** 050501
- [32] Kempe J 2005 Discrete quantum walks hit exponentially faster *Probability Theory and Related Fields* **133** 215
- [33] Kurzyński P and Wójcik A 2011 Discrete-time quantum walk approach to state transfer *Phys. Rev. A* **83** 062315
- [34] Zhan X, Qin H, Bian Z-H, Li J and Xue P 2014 Perfect state transfer and efficient quantum routing: a discrete-time quantum-walk approach *Phys. Rev. A* **90** 012331
- [35] Štefaňák M and Skoupý S 2017 Perfect state transfer by means of discrete-time quantum walk on complete bipartite graphs *Quantum Inf. Process.* **16** 72
- [36] Paparo G D and Martin-Delgado M A 2012 Google in a quantum network *Sci. Rep.* **2** 444
- [37] Paparo G D, Müller M, Comellas F and Martin-Delgado M A 2013 Quantum google in a complex network *Sci. Rep.* **3** 2773
- [38] Chawla P, Mangal R and Chandrashekar C M 2020 Discrete-time quantum walk algorithm for ranking nodes on a network *Quantum Inf. Process.* **19** 158
- [39] Wang K, Shi Y, Xiao L, Wang J, Joglekar Y N and Xue P 2020 Experimental realization of continuous-time quantum walks on directed graphs and their application in PageRank *Optica* **7** 1524
- [40] de Souza L S, de Carvalho J H and Ferreira T A 2019 Quantum walk to train a classical artificial neural network *2019 8th Brazilian Conference on Intelligent Systems (BRACIS), IEEE* 836–41
- [41] de Souza L S, de Carvalho J H A and Ferreira T A E 2022 Classical artificial neural network training using quantum walks as a search procedure *IEEE Trans. Comput.* **71** 378
- [42] Chandrashekar C M, Melville S and Busch T 2014 Single photons in an imperfect array of beam-splitters: Interplay between percolation, backscattering and transient localization *J. Phys. B: At. Mol. Opt. Phys.* **47** 085502
- [43] Chandrashekar C M and Busch T 2015 Quantum percolation and transition point of a directed discrete-time quantum walk *Sci. Rep.* **4** (1) 6583 4
- [44] Chawla P, Ambarish C V and Chandrashekar C M 2019 Quantum percolation in quasicrystals using continuous-time quantum walk *Journal of Physics Communications* **3** 125004
- [45] Childs A M 2009 Universal computation by quantum walk *Phys. Rev. Lett.* **102** 180501
- [46] Lovett N B, Cooper S, Everitt M, Trevers M and Kendon V 2010 Universal quantum computation using the discrete-time quantum walk *Phys. Rev. A* **81** 042330
- [47] Singh S, Chawla P, Sarkar A and Chandrashekar C M 2021 Universal quantum computing using single-particle discrete-time quantum walk *Sci. Rep.* **11** 11551
- [48] Chawla P, Singh S, Agarwal A, Srinivasan S and Chandrashekar C M 2023 Multi-qubit quantum computing using discrete-time quantum walks on closed graphs *Sci. Rep.* **13** 12078
- [49] Nayak A and Vishwanath A 2000 *arXiv* <https://doi.org/10.48550/arXiv.quant-ph/0010117> Oct 2000 Quantum walk on the line
- [50] Childs A M and Goldstone J 2004 Spatial search by quantum walk *Phys. Rev. A* **70** 022314
- [51] Bai L, Hancock E R, Torsello A and Rossi L 2013 A quantum jensen-shannon graph kernel using the continuous-time quantum walk *Graph-Based Representations in Pattern Recognition* vol. 7877 ed D Hutchison *et al* (Springer) 121–31
- [52] Feng Y, Zhou J, Li J, Zhao W, Shi J, Shi R and Li W 2022 SKC-CCCO: an encryption algorithm for quantum group signature *Quantum Inf. Process.* **21** 328
- [53] Mohseni M, Rebentrost P, Lloyd S and Aspuru-Guzik A 2008 Environment-assisted quantum walks in photosynthetic energy transfer *J. Chem. Phys.* **129** 174106
- [54] Ambainis A and Montanaro A 2014 Quantum algorithms for search with wildcards and combinatorial group testing *Quantum Inf. Comput.* **14** 439
- [55] Rhodes M L and Wong T G 2019 Quantum walk search on the complete bipartite graph *Phys. Rev. A* **99** 032301
- [56] Marsh S and Wang J B 2021 Deterministic spatial search using alternating quantum walks *Phys. Rev. A* **104** 022216
- [57] Schnyder A P, Ryu S, Furusaki A and Ludwig A W W 2008 Classification of topological insulators and superconductors in three spatial dimensions *Phys. Rev. B* **78** 195125
- [58] Kitagawa T, Rudner M S, Berg E and Demler E 2010 Exploring topological phases with quantum walks *Phys. Rev. A* **82** 033429
- [59] Kitagawa T 2012 Topological phenomena in quantum walks: elementary introduction to the physics of topological phases *Quantum Inf. Process.* **11** 1107
- [60] Asbóth J K 2012 Symmetries, topological phases, and bound states in the one-dimensional quantum walk *Phys. Rev. B* **86** 195414
- [61] Chandrashekar C M, Obuse H and Busch T 2015 *arXiv* 10.48550/arXiv.1502.00436 Feb 2015 Entanglement properties of localized states in 1D topological quantum walks

- [62] Chandrashekar C M 2013 Two-component Dirac-like Hamiltonian for generating quantum walk on one-, two- and three-dimensional lattices *Sci. Rep.* **3** 2829
- [63] D'Ariano G M and Perinotti P 2014 Derivation of the Dirac equation from principles of information processing *Phys. Rev. A* **90** 062106
- [64] Mallick A and Chandrashekar C M 2016 Dirac cellular automaton from split-step quantum walk *Sci. Rep.* **6** 25779
- [65] Kumar N P, Balu R, Laflamme R and Chandrashekar C M 2018 Bounds on the dynamics of periodic quantum walks and emergence of the gapless and gapped Dirac equation *Phys. Rev. A* **97** 012116
- [66] Garreau J C and Zehnlé V 2020 Analog quantum simulation of the spinor-four Dirac equation with an artificial gauge field *Phys. Rev. A* **101** 053608
- [67] Huerta Alderete C, Singh S, Nguyen N H, Zhu D, Balu R, Monroe C, Chandrashekar C M and Linke N M 2020 Quantum walks and Dirac cellular automata on a programmable trapped-ion quantum computer *Nat. Commun.* **11** 3720
- [68] Mallick A, Mandal S and Chandrashekar C M 2017 Neutrino oscillations in discrete-time quantum walk framework *Eur. Phys. J. C* **77** 1
- [69] Jeong Y-C, Di Franco C, Lim H-T, Kim M and Kim Y-H 2013 Experimental realization of a delayed-choice quantum walk *Nat. Commun.* **4** 2471
- [70] Tang H et al 2018 Experimental two-dimensional quantum walk on a photonic chip *Science Advances* **4** eaat3174
- [71] Gao H, Wang K, Qu D, Lin Q and Xue P 2023 Demonstration of a photonic router via quantum walks *New J. Phys.* **25** 053011
- [72] Chakraborty S, Das A, Mallick A and Chandrashekar C M 2017 Quantum ratchet in disordered quantum walk: quantum ratchet in disordered quantum walk *Ann. Phys.* **529** 1600346
- [73] Szegedy M 2004 Quantum speed-up of Markov chain based algorithms *45th Annual IEEE Symposium on Foundations of Computer Science, IEEE* 32–41
- [74] Chandrashekar C M, Srikanth R and Laflamme R 2008 Optimizing the discrete time quantum walk using a SU(2) coin *Phys. Rev. A* **77** 032326
- [75] Hoyer S and Meyer D A 2009 Faster transport with a directed quantum walk *Phys. Rev. A* **79** 024307
- [76] Inui N, Konishi Y and Konno N 2004 Localization of two-dimensional quantum walks *Phys. Rev. A* **69** 052323
- [77] Chandrashekar C M 2013 *arXiv* 10.48550/arXiv.1212.5984 May 2013 Disorder induced localization and enhancement of entanglement in one- and two-dimensional quantum walks
- [78] Crespi A, Osellame R, Ramponi R, Giovannetti V, Fazio R, Sansoni L, De Nicola F, Sciarrino F and Mataloni P 2013 Anderson localization of entangled photons in an integrated quantum walk *Nat. Photonics* **7** 322
- [79] Fuda T, Funakawa D and Suzuki A 2017 Localization of a multi-dimensional quantum walk with one defect *Quantum Inf. Process.* **16** 203
- [80] Chandrashekar C M, Goyal S K and Banerjee S 2012 Entanglement generation in spatially separated systems using quantum walk *Journal of Quantum Information Science* **02** 15
- [81] Newman M and Watts D 1999 Renormalization group analysis of the small-world network model *Phys. Lett. A* **263** 341
- [82] Barabási A-L and Albert R 1999 Emergence of scaling in random networks *Science* **286** 509