

Implementing the one-dimensional quantum (Hadamard) walk using a Bose-Einstein condensate

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We propose a scheme to implement the simplest and best-studied version of the quantum random walk, the discrete Hadamard walk, in one dimension using a coherent macroscopic sample of ultracold atoms, Bose-Einstein condensate (BEC). Implementation of the quantum walk using a BEC gives access to the familiar quantum phenomena on a macroscopic scale. This paper uses a rf pulse to implement the Hadamard operation (rotation) and stimulated Raman transition technique as a unitary shift operator. The scheme suggests the implementation of the Hadamard operation and unitary shift operator while the BEC is trapped in a long Rayleigh range optical dipole trap. The Hadamard rotation and a unitary shift operator on a BEC prepared in one of the internal states followed by a bit-flip operation, implements one step of the Hadamard walk. To realize a sizable number of steps, the process is iterated without resorting to intermediate measurement. With current dipole trap technology, it should be possible to implement enough steps to experimentally highlight the discrete quantum random walk using a BEC leading to further exploration of quantum random walks and its applications.

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I. INTRODUCTION

Theoretical studies and the experimental possibility to construct quantum computers [1], based on the laws of quantum mechanics has been around for about two and a half decades [2]. The Deutsch-Jozsa algorithm [3], communicated in 1992, suggests that the quantum computers may be capable of solving some computational problems much more efficiently than a classical computer. In 1994, Shor [4] described theoretically a quantum algorithm to factor large semiprimes that was exponentially faster than the best-known classical counterpart. Shor's algorithm initiated active research in the area of quantum information and quantum computation across a broad range of disciplines: quantum physics, computer sciences, mathematics, and engineering. This research has unveiled many new effects that are strikingly different from their classical counterparts. Since Shor's algorithm, Grover in 1997 devised an algorithm [5], which can, in principle, search an unsorted database quadratically faster than any classical algorithm. Yet there is still a search for new quantum search algorithms, which can be practically realized. In this direction, adapting a known classical algorithm has also been considered as an option.

The random walk, which has found applications in many fields [6], is one of the aspects of information theory and it has played a very prominent role in classical computation. Markov chain simulation has emerged as a powerful algorithmic tool [7] and many other classical algorithms are based on random walks. It is believed that exploring quantum random walks allows, in a similar way, a search for new quantum algorithms. Quantum random walks [8] have been investigated by a number of groups and several algorithms have been proposed [9–12]. To implement such an algorithm in a physical system, a quantum walker moving in a position and momentum space has to be first realized. Some setups for one-dimensional realizations have been proposed and analyzed, including trapped ions [13] and neutral atoms in optical lattice [14]. One- and two-dimensional quantum

walks in an array of optical traps for neutral atoms has also been analyzed in Ref. [15].

This paper proposes a scheme to experimentally implement a discrete quantum (Hadamard) walk using a coherent macroscopic sample of ultracold atoms, a Bose-Einstein condensate (BEC), in one dimension. The implementation of the quantum walk using a BEC gives access to the quantum phenomena on a macroscopic scale. With the current BEC trapping technique, it should be possible to implement enough steps to experimentally highlight the quantum walk by generating spatially large-scale entanglement and lead to the further exploration of the quantum walk and its applications. To implement the quantum walk, the BEC trapped in a far detuned optical dipole trap with a long Rayleigh range [16] is made to evolve into the quantum superposition (Schrödinger cat) state of the two trappable states by applying the Hadamard operation. The BEC in the Schrödinger cat state is then subjected to a unitary shift operator to translate the condensate in the axial direction of the optical trap. Thereafter, a compensatory bit-flip operator is applied.

This paper uses rf pulses to implement the Hadamard rotation and the bit-flip operation, stimulated Raman transition technique as a unitary shift operator to impart a well-defined momentum to translate the BEC. This method of applying a unitary shift operator by subjecting it to optical Raman pulses to drive transitions between two internal trappable states giving the BEC a well-defined momentum in an axial direction of the trap can be called *stimulated Raman kicks*. The Hadamard operation followed by a unitary shift operation and bit-flip operation implements one step of the quantum (Hadamard) walk. A sizable number of steps can be implemented by iterating the process without resorting to the intermediate measurement.

The paper is organized as follows. Section II has a brief introduction to the quantum random walk. Section III discusses the quantum walk using the BEC. The creation of a Schrödinger cat state of the BEC, stimulated Raman kicks, used to implement a unitary operator and the compensatory bit flip is discussed in Secs. III A–III C, respectively. Section

IV discusses the physical setup required for the implementation of quantum walks using the BEC. Section V discusses decoherence and physical limitations and Sec. VI concludes with the summary.

II. QUANTUM RANDOM WALK

Quantum random walks are the counterpart of classical random walks for particles, which cannot be precisely localized due to quantum uncertainties. The word quantum random walk, for the first time, was coined in 1993 by Aharonov, Davidovich, and Zagury [17]. In the one-dimensional classical random walk, making a step of a given length to the left or right for a particle is described in terms of probabilities. On the other hand, in the quantum random walk it is described in terms of probability amplitudes. In an unbiased one-dimensional classical random walk, with the particle initially at x_0 , it evolves in such a way that at each step, the particle moves with probability 1/2 one step to the left and with probability 1/2 one step to the right. In a quantum-mechanical analog, the state of the particle evolves at each step into a coherent superposition of moving one step to the right and one step to the left with an equal probability amplitude.

Unlike the classical random walk, two degrees of freedom are required for quantum random walks, internal state (superposition), which is called *coin* Hilbert space \mathcal{H}_c (quantum coin), and the *position* Hilbert space \mathcal{H}_p . Imagine a coin Hilbert space \mathcal{H}_c of a particle on a line spanned by two basis states $|0\rangle$ and $|1\rangle$ and the position Hilbert space \mathcal{H}_p spanned by a basis state $|x\rangle : x = \mathbf{Z}$. The state of the total system is in the space $\mathcal{H} = \mathcal{H}_c \otimes \mathcal{H}_p$. The internal state of the particle determines the direction of the particle movement when the unitary shift operator U is applied on the particle,

$$U = |0\rangle\langle 0| \otimes \sum |x-1\rangle\langle x| + |1\rangle\langle 1| \otimes \sum |x+1\rangle\langle x|. \quad (1)$$

If the internal state is $|0\rangle$, the particle moves to the left, while it moves to the right if the internal state is $|1\rangle$, i.e.,

$$U|0\rangle \otimes |x\rangle = |0\rangle \otimes |x-1\rangle$$

and

$$U|1\rangle \otimes |x\rangle = |1\rangle \otimes |x+1\rangle. \quad (2)$$

Each step of the quantum (Hadamard) walk is composed of the Hadamard operation (rotation) H ,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad (3)$$

on the particle to bring them to a superposition state with equal probability, such that,

$$(H \otimes \mathbb{1})|0, x\rangle = \frac{1}{\sqrt{2}}[|0, x\rangle + |1, x\rangle]$$

and

$$(H \otimes \mathbb{1})|1, x\rangle = \frac{1}{\sqrt{2}}[|0, x\rangle - |1, x\rangle] \quad (4)$$

followed by a unitary shift operation, U , which moves the particle in the superposition of the position space. If the particle is prepared in the superposition state then each step of the quantum walk consists of a unitary shift operator followed by a Hadamard operation.

The probability amplitude distribution arising from the iterated application of $W = U(H \otimes \mathbb{1})$ is significantly different from the distribution of the classical walk after the first two steps [8]. If the coin initially is in a suitable superposition of $|0\rangle$ and $|1\rangle$, then the probability amplitude distribution after n steps of the quantum walk will have two maxima symmetrically displaced from the starting point. The variance of the quantum version grows quadratically with the number of steps n , $\sigma^2 \propto n^2$ compared to $\sigma^2 \propto n$ for the classical walk.

III. QUANTUM WALK USING THE BOSE-EINSTEIN CONDENSATE

As discussed in Sec. II, two degree of freedom, the coin Hilbert space \mathcal{H}_c , and the position Hilbert space \mathcal{H}_p are required to implement the quantum walk. A state of the BEC formed from the atoms in one of the hyperfine states $|0\rangle$ or $|1\rangle$ can be represented as $|0_{BEC}\rangle$ or $|1_{BEC}\rangle$ (the state of N condensed atoms). The BEC formed is then transferred to an optical dipole trap with long Rayleigh range Z_R . With the appropriate choice of power and beam waist, ω_0 [18] of the trapping beam, the condensate can remain trapped at any point within the distance $\pm Z$ from the focal point in the axial direction of the beam [19]. The BEC in the optical trap is then subjected to evolve into the superposition (Schrödinger cat) state $\frac{1}{\sqrt{2}}[|0_{BEC}\rangle + i|1_{BEC}\rangle]$ (Sec. III A) by applying the Hadamard rotation. The coin Hilbert space \mathcal{H}_c can then be defined to be spanned by the two internal trappable states $|0_{BEC}\rangle$ and $|1_{BEC}\rangle$ of the BEC. The BEC in the superposition state is spatially translated by applying a unitary shift operator, then the position Hilbert space \mathcal{H}_p is spanned by the position of the BEC in the long Rayleigh range optical trap and is augmented by the coin Hilbert space \mathcal{H}_c .

The position of the BEC trapped in the center of the optical trap is described by a wave packet $|\Psi_{x_0}\rangle$ localized around a position x_0 , i.e., the function $\langle x | \Psi_{x_0} \rangle$ corresponds to a wave packet centered around x_0 . When the BEC is subjected to evolve into the superposition ($|S\rangle = [a|0_{BEC}\rangle + b|1_{BEC}\rangle]$) of the eigenstates $|0_{BEC}\rangle$ and $|1_{BEC}\rangle$, its wave function is written as

$$|\Psi_{in}\rangle = \frac{1}{\sqrt{2}}[|0_{BEC}\rangle + |1_{BEC}\rangle] \otimes |\Psi_{x_0}\rangle, \quad (5)$$

where $|a|^2 + |b|^2 = 1$ and for the symmetric superposition $a = b = \frac{1}{\sqrt{2}}$. Once the BEC is in the superposition state, a unitary shift operator, *stimulated Raman kick* (Sec. III B), corresponding to one step length l is applied;

$$U' = (\hat{X} \otimes \mathbb{I}) \exp(-2iS_z \otimes Pl), \quad (6)$$

P being the momentum operator and S_z the operator corresponding to the step of length l . Step length l is chosen to be less than the spatial width of the BEC wave packet $|\Psi_{x_0}\rangle$.

The application of the unitary shift operator (U) on the wave function of Eq. (5) spatially entangles the position and the coin space and implements the quantum walk

$$U'|\Psi_{in}\rangle = \frac{1}{\sqrt{2}}[|1_{BEC}\rangle \otimes e^{-iPl} + |0_{BEC}\rangle \otimes e^{iPl}]|\Psi_{x_0}\rangle, \quad (7)$$

where the wave packet is centered around $x_0 \pm l$. Note that the values in the coin space have been flipped. This is corrected by applying a compensating bit flip on the BEC. After each compensated unitary shift operator, $U \equiv (\hat{X} \otimes \mathbb{I})U'$, the BEC settles down in one of the internal states $|0_{BEC}\rangle$ or $|1_{BEC}\rangle$. Therefore, a Hadamard rotation is applied to make the BEC evolve into a superposition state. A Hadamard rotation followed by a unitary shift operator implements one step of the Hadamard walk. To realize the large number of steps the process is iterated without resorting to intermediate measurement in the long Rayleigh-range optical dipole trap.

A. Macroscopic cat state in the Bose-Einstein condensate

Various schemes have been proposed for producing macroscopic superposition or Schrödinger cat states in BECs [20–22]. The scheme described by Cirac *et al.* in Ref. [20] shows that if the two species are Josephson coupled, then in certain parameter regimes the ground state of the Hamiltonian is a superposition of two states involving a particle number imbalance between the two species. Such a state represents a superposition of two states, which are macroscopically (or mesoscopically) distinguishable, and hence can be called a Schrödinger cat state. Ruostekoski *et al.* [21] have also shown that such states can be created by a mechanism involving the coherent scattering of far-detuned light fields and it neglects the collisional interactions between particles. The major drawback of the above schemes is that the time needed to evolve to a cat state can be rather long, and thus problems due to decoherence would be greatly increased. In Ref. [20] the macroscopic superposition is produced by the normal dynamic evolution of the system. Gordon *et al.* [22] have proposed a scheme in which the superposition is produced by an adiabatic transfer of the ground state of the Josephson-coupling Hamiltonian, that is, after the initial state preparation the Josephson-coupling is turned on for some amount of time and turned off. The resulting modified quantum state is a Schrödinger cat state. But, the production of such a state using this scheme [22] involves considerable experimental difficulty.

The ideal scheme to demonstrate the quantum walk is to confine the BEC that has an attractive interaction between atoms in two hyperfine levels $|0\rangle$ and $|1\rangle$ in a single optical potential well. The Hadamard operation (rotation) is applied on the BEC in the potential well to transfer (or rotate) the atoms part of the way between states $|0\rangle$ and $|1\rangle$ using a resonant rf pulse [23] of duration τ and detuning Δ from the

rf resonance [24]. The resonance rf pulse couples the atomic hyperfine states $|0\rangle$ and $|1\rangle$ with a coupling matrix element $\hbar\omega_R/2$, where ω_R is the Rabi frequency and the duration of pulse is much shorter than the self-dynamics of the BEC. The amplitude of these states evolves according to the Schrödinger equation,

$$i\hbar \frac{d}{d\tau} \begin{pmatrix} a \\ b \end{pmatrix} = \hbar \begin{pmatrix} 0 & \omega_R/2 \\ \omega_R/2 & \Delta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}. \quad (8)$$

At this stage, each atom evolves into the superposition state $a(\tau)|0\rangle + b(\tau)|1\rangle$, with $a(\tau) = \cos(\omega_R\tau/2)$ and $b(\tau) = \sin(\omega_R\tau/2)$ for detuning $\Delta=0$. The N -particle wave function of the BEC is a product of the single-particle superpositions of $|0\rangle$ and $|1\rangle$, that is, it is still a microscopic superposition and is given by

$$\begin{aligned} & [a(\tau)|0\rangle + b(\tau)|1\rangle]^N \\ &= \sum_{n=0}^N \sqrt{\frac{N!}{n!(N-n)!}} a(\tau)^{N-n} b(\tau)^n |N-n, n\rangle, \end{aligned} \quad (9)$$

where $|N-n, n\rangle$ is the state with $N-n$ atoms in state $|0\rangle$ and n atoms in state $|1\rangle$. The individual atoms in the superposition state interact among themselves. Interatomic interactions, which provide nonlinear terms through binary collision as seen from the viewpoint of single-particle dynamics helps in generating highly entangled many-body states [25,26]. The attractive interatomic interaction between atoms in the superposition state induce entanglement between atoms. Thus Eq. (9) evolves into the macroscopic superposition state in which all atoms are simultaneously in level $|0\rangle$ and level $|1\rangle$ [20,27,28],

$$[a(\tau)|0\rangle + b(\tau)|1\rangle]^N = a(\tau)^N |N, 0\rangle + b(\tau)^N |0, N\rangle, \quad (10)$$

where $|N, 0\rangle = |0_{BEC}\rangle$ and $|0, N\rangle = |1_{BEC}\rangle$. Symmetric probability distribution $a(\tau) = b(\tau) = \frac{1}{\sqrt{2}}$ can be obtained by carefully choosing ω_R and τ .

B. Unitary operator—Stimulated Raman kicks

A unitary shift (controlled-shift) operation U is applied on the BEC to spatially entangle the position and the coin space and implement the quantum walk. Various schemes have been worked out to give momentum kick to ultracold atoms in a trap [28,29]. Mewes *et al.* [28] had developed a technique where coherent rf-induced transitions were used to change the internal state of the atoms in the magnetic trap from a trapped to an untrapped state and thus displacing the untrapped atoms from the trap. This method, however, did not allow the direction of the output-coupled atoms to be chosen. Hagley *et al.* [29] developed a technique to extract sodium atoms from the trapped BEC by using a stimulated Raman process between magnetic sublevels to have a control on the direction of the fraction of the outgoing BEC and was used to extract atom lasers from the BEC.

A stimulated Raman process can also be used to drive the transition between two optically trappable states of the atom $|0\rangle$ and $|1\rangle$ using the virtual state $|e\rangle$ as an intermediary state, and impart a well-defined momentum to spatially translate

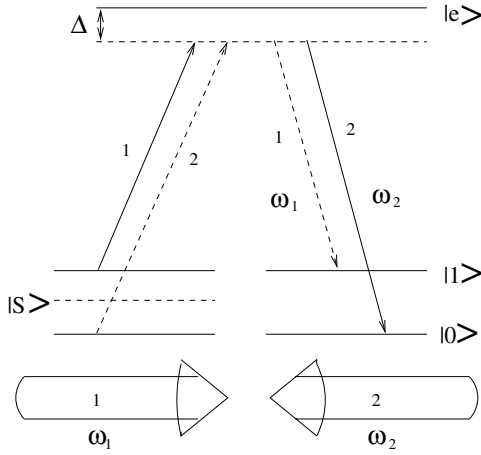


FIG. 1. Light field configuration for a stimulated Raman transition process to give directional momentum kick. Δ is the detuning of the laser from its transition frequency. $|S\rangle$ signifies $(1/\sqrt{2}) \times (|0\rangle \pm |1\rangle)$.

the atoms in the coherent state (BEC). A unitary shift operation U' thus can be applied on the atom in the BEC using a stimulated Raman process. A pair of counterpropagating laser beams 1 and 2 (Fig. 1) with frequency ω_1 and ω_2 and wave vectors k_1 and k_2 is applied on the BEC for a $2N$ -photon transition time (N is the number of atoms in the BEC) to implement one unitary shift operation. These beams are configured to propagate along the axial direction of the optical dipole trap. A stimulated Raman transition occurs when an atom changes its state by coherently exchanging photons between the two laser fields—absorption of the photon from laser field 1 and stimulated emission into laser field 2 or by absorption from field 2 and stimulated emission into field 1. The BEC initially in eigenstates of the atom $|0\rangle$ ($|1\rangle$) can absorb the photon from field 1 (2) and reemit the photon into field 2 (1). This inelastic stimulated Raman scattering process imparts well-defined momentum on the coherent atoms,

$$\mathbf{P} = \hbar(k_1 - k_2) = \hbar \delta z \quad (11)$$

to the left during $|0\rangle \xrightarrow{\omega_1} |e\rangle \xrightarrow{\omega_2} |1\rangle$ and

$$\mathbf{P} = \hbar(k_2 - k_1) = -\hbar \delta z \quad (12)$$

to the right during $|1\rangle \xrightarrow{\omega_2} |e\rangle \xrightarrow{\omega_1} |0\rangle$, where

$$|k_1 - k_2| = |k_2 - k_1| = \delta.$$

Thus, conditioned to being in coin state $|0\rangle$ ($|1\rangle$), the atoms in the BEC receive a momentum kick (shift) $\hbar \delta z$ ($-\hbar \delta z$). This process of imparting momentum is called stimulated Raman kick and can be analyzed as a photon absorption and stimulated emission between three bare states $|0\rangle$, $|1\rangle$, and $|e\rangle$ driven by two monochromatic light fields. The matrix element for photon absorption and stimulated emission during each stimulated Raman kick can be written as

$$\langle n_{k_1} - 1, n_{k_2} + 1, 1 | \hat{H}_{I_a} | n_{k_1}, n_{k_2}, 0 \rangle \quad (13)$$

during $|0\rangle \xrightarrow{\omega_1} |e\rangle \xrightarrow{\omega_2} |1\rangle$ and

$$\langle n_{k_1} + 1, n_{k_2} - 1, 0 | \hat{H}_{I_b} | n_{k_1}, n_{k_2}, 1 \rangle \quad (14)$$

during $|1\rangle \xrightarrow{\omega_2} |e\rangle \xrightarrow{\omega_1} |0\rangle$. H_{I_a} and H_{I_b} are the interaction Hamiltonians (electric dipole Hamiltonians). The field causing the transitions between bare states $|0\rangle$ and $|e\rangle$ is detuned from resonance by Δ_1 and has a constant dipole matrix element V_1 and phase φ_1 . The field causing the transitions between bare states $|e\rangle$ and $|1\rangle$ is detuned from resonance by Δ_2 and has a constant dipole matrix element V_2 and phase φ_2 . The time-independent Hamiltonian in the rotating-wave approximation for this system can be written in terms of projection operations as

$$\begin{aligned} \hat{H}_{I_a} = & \hbar \Delta_1 |e\rangle \langle e| + \hbar (\Delta_1 + \Delta_2) |1\rangle \langle 1| - \frac{\hbar V_1}{2} [|e\rangle \langle 0| \exp(-i\varphi_1) \\ & + |0\rangle \langle e| \exp(i\varphi_1)] - \frac{\hbar V_2}{2} [|e\rangle \langle 1| \exp(i\varphi_2) \\ & + |1\rangle \langle e| \exp(-i\varphi_2)]. \end{aligned} \quad (15)$$

In the above interaction, picture the energy of the bare state $|0\rangle$ is chosen to be zero.

$$\begin{aligned} \hat{H}_{I_b} = & \hbar \Delta_2 |e\rangle \langle e| + \hbar (\Delta_1 + \Delta_2) |0\rangle \langle 0| - \frac{\hbar V_2}{2} [|e\rangle \langle 1| \exp(-i\varphi_2) \\ & + |1\rangle \langle e| \exp(i\varphi_2)] - \frac{\hbar V_1}{2} [|e\rangle \langle 0| \exp(i\varphi_1) + |0\rangle \\ & \times \langle e| \exp(i\varphi_1)]. \end{aligned} \quad (16)$$

In the above interaction picture the energy of the bare state of the atom $|1\rangle$ is chosen to be zero. $V_i = \frac{\mu_i E_0}{\hbar}$ for $i=1, 2$. μ_i is the dipole operator associated with the states $|0\rangle, |e\rangle$ and $|e\rangle, |1\rangle$ and E_0 is the electric field of the laser beam.

With the appropriate choice of V , Δ , and φ , the probability of being in bare state $|1\rangle$ ($|0\rangle$) depending on the starting state) after time t can be maximized to be close to one, where t is the time required for one stimulated Raman kick. As discussed in Sec. III A, after every stimulated Raman kick on atoms due to the attractive interaction between atoms, they evolve into the BEC state.

C. Bit-flip operation

The coin state of the atoms in the BEC $|0\rangle$ ($|1\rangle$) after stimulated Raman kick flips to $|1\rangle$ ($|0\rangle$). This is reversed via a compensatory bit flip implemented using a resonant rf (π) pulse of duration τ_{bf} and detuning Δ from the resonance. The amplitude of these states evolves, as before, according to Eq. (8).

IV. PHYSICAL SETUP FOR THE IMPLEMENTATION OF THE QUANTUM WALK USING THE BOSE-EINSTEIN CONDENSATE

A magnetic trap technique [30] played a major role in the first formation and early experiments on the BEC. But the magnetic trap has a limitation to trap and manipulate atoms only of certain sublevels. For ^{87}Rb atoms, the $|F=2, m_f=2\rangle$ state can be confined in the magnetic trap whereas $|F=1, m_f=1\rangle$ cannot be confined using the magnetic trap [31]. Under appropriate conditions, the dipole trapping mechanism is independent of the particular sublevel of the electronic ground state. The internal ground state can thus be fully exploited using an optical dipole trap technique and be widely used for various experiments [32]. Today the BEC from bosonic atoms has been very consistently formed and manipulated using various configurations of magnetic and optical traps, an all-optical dipole trap technique has also been developed [33].

A BEC formed in one of their internal states (eigenstates) $|0\rangle$ or $|1\rangle$, using any of the techniques [30,33] mentioned above, is transferred to a far detuned, long Rayleigh range (Z_R) optical dipole trap. A sizable number of steps of the Hadamard walk can be implemented within the axial range $\pm Z$ without decoherence in a long Rayleigh-range trap. Reference [19] has calculated numerical values of potential depth, power is required to trap ^{87}Rb atoms at distance $Z = x_n$ from the focal point of the trapping beam (after compensating for gravity) using light fields of various frequency and beam waists ω_0 . In the same way one can work out required power and beam waists to trap ultracold atoms at a distance Z from the focal point for different species of atoms using laser fields of different detuning from the resonance.

Once the BEC is transferred into the optical dipole trap, a resonant rf pulse (Hadamard rotation) of duration τ and detuning Δ is applied to make it evolve into the Schrödinger cat state. The Schrödinger cat state for the ^{87}Rb BEC trapped in one of the states $|0\rangle = |F=1, m_f=1\rangle$ or $|1\rangle = |F=2, m_f=2\rangle$, can be realized by applying a rf pulse, fast laser pulse, a standard Raman pulse, or microwave techniques.

The BEC in the Schrödinger cat state is then subjected to a pair of counterpropagating beams to implement the stimulated Raman kick for duration t , $2N$ photon transition time (N being the number of photons). After implementing the stimulated Raman kick the BEC is left to translate for a duration P/ml , the time taken by the condensate to move distance l during or after which the rf π pulse is applied. This process of applying a rf pulse (Hadamard rotation) and a stimulated Raman kick (shift), followed by a compensatory rf pulse (bit flip) is iterated throughout the trapping range to realize a large number of steps (Fig. 2).

Measuring the one-dimensional quantum walk probability distribution

After n steps of the one-dimensional Hadamard walk the superposition in position space is made to collapse by applying multiple microtraps to confine and locate the position of the BEC. The multiple microtraps in a line are created using the known technique [34,35] (Fig. 3) and are switched on,

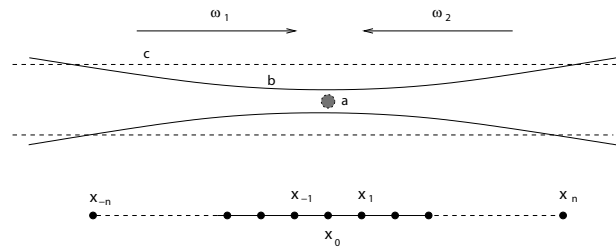


FIG. 2. Physical setup to implement the quantum walk using the BEC. In the figure **a** is a BEC, **b** is a dipole trap with a long Rayleigh range, and **c** is the counterpropagating laser beam, ω_1 and ω_2 used to implement the stimulated Raman kick, unitary operator. A Hadamard rotation and Raman pulse are applied throughout the trap region. The stimulated Raman pulse and the rf pulse are applied alternatively to realize the Hadamard walk.

removing the long Rayleigh range optical trap simultaneously after a time interval of T .

$$T = nt + (n\tau - 1) + n + nP/ml + (n\tau_{\text{bf}} - 1).$$

T is the time taken by the BEC to travel n quantum steps each of distance l in a line. The time t is the $2N$ -photon transition time required to implement one unitary operation, τ is the time duration of the pulse used to bring the BEC to the superposition state, P/ml is the time taken by the BEC to move one step distance l with momentum P , and τ_{bf} is the time duration required to implement bit flip. Note that this time can be eliminated if the bit flip is performed during the translation of the BEC.

The spacing between each potential well is designed to be equal to the distance l . The condensate, which has undergone the Hadamard walk, is confined in one of the traps of the multiple microtrap when it is turned on. Fluorescence measurement is performed on the condensate in the microtrap to identify the final position of the BEC on which the Hadamard walk has been implemented. By repeating the experiment for a fixed number of steps (time T), the probability distribution of the position of the BEC after n steps of the Hadamard walk can be obtained.

V. DECOHERENCE AND PHYSICAL LIMITATIONS

The decoherence of the BEC state also leads to the decoherence of the quantum walk. When the atoms in the trap are not coherent they no longer follow the coherent absorption

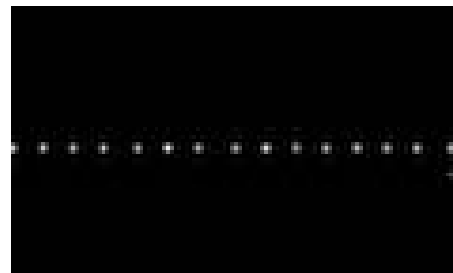


FIG. 3. Multiple microtraps to confine the BEC after it implements a considerable number of steps of the quantum walk.

and stimulated emission of light. Some atoms absorb light field 1 and emit light field 2 and bring atoms to state $|1\rangle$ and some absorb light field 2 and emit light field 1 and bring atoms to state $|0\rangle$, displacing atoms in both directions in space giving no signature of displacement in the superposition of position space. This will contribute to collision and heating, and finally, atoms escape out of the trap. The number of quantum steps that can be implemented using the BEC without decoherence depends mainly on (a) the Rayleigh range, as the BEC moves away from the trap center the width of the BEC wave packet increases and contributes to the internal heating of the atoms. Beyond a certain distance x_n from the trap center, atoms in the trap decohere resulting in the collapse of the quantum behavior. (b) The stimulated Raman kick and rf pulse used to implement the Hadamard walk also contribute to the internal heating and decoherence of the atoms after a few iterations of the Hadamard walk. To implement a Hadamard walk, the spatial width of the particle has to be larger than the step length l [16] and hence, l for the Hadamard walk using the BEC is fixed depending on the spatial width of the BEC used to implement the quantum random walk. However, decoherence of the Hadamard walk does not happen even if some fraction atoms are lost from the BEC (and also until the width of the wave packet is larger than the step length).

With the careful selection of beam waist and laser power one can have a trap with a long Rayleigh range Z_R [19] so that the one-dimensional quantum walk can be implemented in a line of close to one centimeter.

VI. SUMMARY AND CONCLUSION

This paper proposes the use of the BEC in an optical dipole trap to implement the quantum (Hadamard) walk in one dimension. The quantum walk is performed in position space by periodically applying the stimulated Raman kick and manipulating the internal state of the BEC by using a rf pulse, Raman pulse, or a microwave technique. After n steps the multiple microtrap is applied to confine the BEC and fluorescence measurement is done to locate its final position. To implement the Hadamard walk the width of the wave packet has to be larger than the step length l and the width of the BEC wave packet is significantly larger than the other quantum particles suggested to implement the Hadamard walk. This is one of the major advantages of using the BEC to implement the Hadamard walk. With the presently available efficient trapping technology, realizing the quantum random walk using the BEC in a length of close to one centimeter (few 100 steps) is expected. Realizing the quantum random walk using the BEC will be of particular interest as the BEC is a macroscopic wave packet. This will give access to understanding the phenomena in a macroscopic scale.

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